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UNCERTAINTY ANALYSIS OF RAINFALL-RUNOFF PREDICTIONS FOR A SMALL URBANIZED BASIN

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1. INTRODUCTION
1.1. Overview of the problem

Present cities suffer more and more frequently from flooding and associated water qualitative problems due to climate and social changes (e.g. Rosso, Rulli 2002; Ott, Uhlenbrook 2004; Shepherd 2005; Schaefli et al. 2011). Thus, flooding is regarded nowadays as the most damaging natural hazard (Ohl, Tapsell 2000; Opperman et al. 2009).

Climate changes relevant for hydrological processes are generally identified with changes in air temperature and precipitation (volume and intensity) (Blöschl, Montanari 2010). Social changes are identified with the globally observed development of cities (urbanization) and a human trend towards living in floodplains; areas periodically inundated by river overflows (Junk et al. 1989). Results are land-use changes such as deforestation, civil constructions, landscape replacements i.e. substituting natural and semi-natural (permeable) surfaces with artificial (impermeable) ones. As a consequence of land-scapes modifications, frequent but moderate flooding of previously rural areas is avoided, and rare but catastrophic flooding of currently urbanized or industrialized areas is exacerbated (Werner, McNamara 2007). Also, land-use changes are usually followed by decreasing chemical and ecological water quality resulting from wash-off of polluted surfaces during rainfall events (Obropta, Kardos 2007; Dietz, Clausen 2008). Most of pollutants introduced into water with stormwater are associated with sediment particles (Horowitz, Stephens 2008).

As a consequence of changing conditions (climate and social), flood risk and associated water quality problems are dramatically increasing in many parts of the world (Milly et al. 2002; Rosso, Rulli 2002; Di Baldassarre et al. 2010). Especially Central-Eastern Europe is exposed to a high risk of future urban development

(UNFPA 2007), due to European Union (EU) enlargement and its economic and social consequences. Because the consequences of potential flooding pose a greater threat to population and infrastructure in urbanized than in rural areas (Yang et al. 2010), urbanized areas are also of a higher concern in the hydrological community.

To assess the effects of changing basin conditions and propose mitigation strategies, urban planners and decision makers have to rely on hydrological model predictions from a design storm, long-term precipitation records or climate scenarios. This is not a trivial task. Most of all, because a hydrological model represents only a simplification of a real basin (Ratto et al. 2007). Due to its restricted structure tied up with its parameters, its ability to model the observed process, e.g. rainfall-runoff (RR), is limited. A calibration of model parameters with observed input-output data usually improves model predictions, resulting in model parameters which better reproduce the observed patterns. Unfortunately, hydrological and meteorological observations and basin data are not generally available (Sivapalan 2003). This results in problems with model calibration and with providing reliable predictions. Thus, hydrological models suffer from i) input uncertainty, ii) structural limitations, iii) parameter uncertainty, and iv) output uncertainty (Sikorska et al. 2013).

In this regard, modelling in small urbanized basins, SUBs, is especially difficult. Hereafter, SUBs are defined as basins in which the ratio of urban sites has been significantly increased over time. The difficulties of hydrological modelling in SUBs have two main reasons. First, the influence of possible future climate and social changes on hydrological conditions in SUBs are hardly predictable due to a very small contributing area. And second, sufficient observed data are usually not available for SUBs (Sikorska et al. 2012a). In addition, because of a small contributing area and a very fast basin response to rainfall, frequent data records are required, which would be too costly to be implemented in the SUBs.

Consequently, rational urban water management should ideally consider not only the most probable prediction in terms of classical (deterministic) modelling but also the associated uncertainties (Krzysztofowicz 1983; Murphy 1991; Krzysztofowicz 1999). Although providing exclusively qualitative information on prediction uncertainty (certain? / uncertain?) may take the possible risk into consideration, it cannot be practically introduced into water management. To this end, uncertainty of model predictions must be quantified (how much uncertain?). To this end, diverse uncertainty sources need to be formally (implicitly or explicitly) described. A formal description of the model structure errors is the most challenging because errors of hydrological models are usually strongly autocorrelated (e.g. Romanowicz et al. 1994; Kuczera et al. 2006; Sikorska et al. 2012a, b). This can be explained by the memory effect of the basin. Thus, the classical Gaussian model error, with identically (normally) and independently distributed (i.i.d.) errors, does not hold for hydrological models. A promising alternative was recently proposed by Yang et al. (2007, 2008), who investigated the autoregressive lumped error model to lump diverse uncertainty sources into a single error term. This error model accounts for the correlation apparent between hydrological model errors. Yet, this error model has not been widely recognized so far, especially in application to SUBs. In a similar fashion, the Gaussian model error does not hold for errors of the input variable, which usually is the rainfall for RR models. Such errors are usually time-dependent,

which can be explained by the variability in the intensity of rainfall fields, changing between the rainfall events. To capture this diversity in the input variable, it must be described by time-dependent parameters. Recently, a promising approach of rainfall multipliers has been proposed by Kavetski et al. (2006a, b) but has not been commonly applied in hydrology until now, especially in the SUBs. Furthermore, it is not clear how to formally describe errors in the output variable, typically streamflow for RR models, and more importantly how to include these errors into hydrological model predictions.

Given aforementioned considerations, a feasible method to formally acknowledge diverse sources of the predictive uncertainty (PU) in hydrological modelling and to quantify the aggregated total PU is needed.

This work provides a formal approach for uncertainty analysis (UA) of rainfall--runoff (RR) predictions, particularly in small urbanized basins (SUBs). The term 'formality' implies that the computed uncertainty should be statistically correct, probabilistically interpretable and yet practically feasible. The proposed uncertainty analysis (UA) approach relies on Bayesian statistics as it proved to be conceptually more satisfactory than other uncertainty analysis approaches. To account for uncertainty in RR modelling, a basin's RR process is modelled as a stochastic process, which may evolve in many directions. To include the uncertainty in the model predictions, a traditional deterministic hydrological model, giving a single output, is combined with an error term. The proposed UA is innovative in two ways. First, it allows one to formally quantify the predictive uncertainty (PU) of RR by applying the state of-art Bayesian inference combined with the autoregressive error model to capture errors of model structural deficits. Second, it allows one to relatively assess the contribution of four main acknowledged uncertainty sources. This was not done before for SUBs.

The main concern is given to rainfall-driven floods induced by stormwater (SW) runoffs. Thus, quantitative variables such as streamflow and water level are of interest because these are most relevant for flood risk studies.

The PU of RR predictions is investigated by means of two practical case studies, in which the usefulness of UA is demonstrated in its application to:
1. streamflow prediction (1 variable, 1 model),
2. water level prediction (1 variable, 2 submodels).

The UA approach is tested on chosen hydrological models in a small experimental basin in Warsaw, Sluzew Creek, Poland. Because the Sluzew Creek basin, like a typical SUB, is not covered with a continuous monitoring program, an experimental campaign has been performed in order to obtain sufficient data for the desired analysis.

Although the UA approach was applied to SUB, it is independent from the experimental basin and the hydrological models. Therefore predictions within diverse models on basins with diverse land-use types can also be evaluated. Moreover, the approach is independent from analyzed hydrological characteristics and may be also applied to model water quality parameters.

1.2. Research questions

1. How can the total predictive uncertainty (PU) of RR predictions in SUBs be practically quantified?
2. How can autocorrelated errors of hydrological models be practically and formally acknowledged to provide reliable uncertainty predictions? Is the autoregressive lumped error model sufficient in describing errors of hydrological models for SUBs?
3. Is a rainfall multipliers approach with time-dependent parameters sufficient enough to capture and describe the variability in precipitation, as an input into RR models for SUBs?
4. How can output uncertainty of RR models, typically represented by measured streamflow, be formally acknowledged to investigate its influence on the total PU?
5. How can the total PU be incorporated into practical applications and can it be useful? How can the total PU be reduced and the model predictions improved?
6. How can consequences of future (climate or social) changes be predicted in SUBs with the scarce input-output variables data available? Are conceptual models with reduced complexity reliable in providing such predictions?
7. What is the influence of the monitoring and hydrological data situation in Central-Eastern Europe on RR modelling in SUBs?
8. What are the main sources of the total PU in RR predictions in the Sluzew Creek basin?

2. RAINFALL-RUNOFF MODELLING IN SMALL URBANIZED BASINS (SUB)
2.1. Rainfall-driven flooding in SUB

2.1.1. Hydrological moddeling and its challenges

Hydrological modelling is an important tool to simulate real system (i.e. basin) response when statistical methods which require long-term data series cannot be applied. This includes such applications as predicting basin response under unknown future conditions in changing environment (climate or social), prolonging observed records or generating synthetic data (Ciepielowski, Dąbkowski 2006). To this end, a physical basin must be substituted with a conceptual version i.e. model that imitates its behaviour (Wagener et al. 2003, Wagener, Montanari 2011), see Fig. 2.1. Such a hydrological model M is usually constructed as a perceptual and conceptual hydrologists' belief and understanding of a physical basin behaviour. This belief is typically supported by extensive empirical data. Any model is represented through linking model parameters (θ^M) by mathematical relationships with the model input (X) and model output (y) (Wagener et al. 2004) and can be described:

$$y = M(X, \theta^M) \tag{2.1}$$

Following Wagener et al. (2004), all hydrological models are lumped at some level because their parameters are simplified to represent a behaviour of a heteroge-
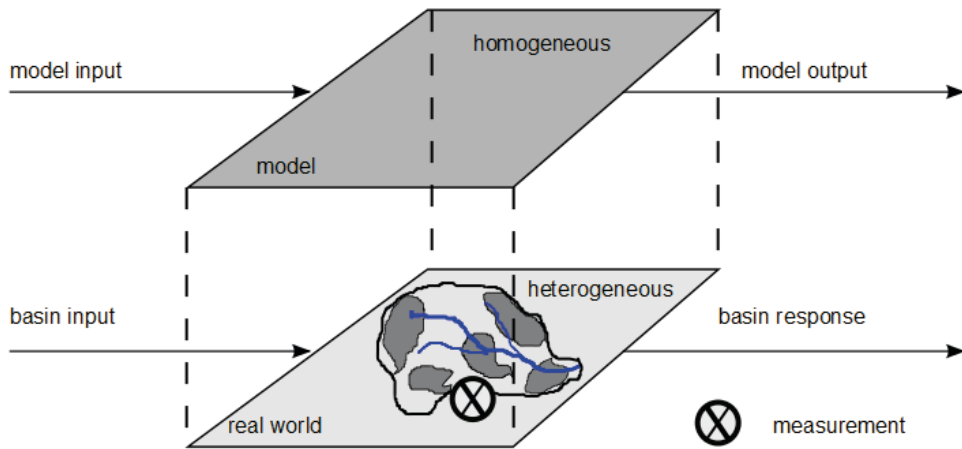
Fig. 2.1. Basin versus model; reproduced from Wagener et al. 2004

neous real world system as a homogeneous model's cell (Fig. 2.1). A model's cell represents the simplest model structure which is not further decomposed and has generalized characteristics over time and space, e.g. basin, sub-basin, hydro-layer, etc. Such simplification is necessary due to the human inability to observe all factors of basin patterns in sufficient enough details (as geology, hydrology, meteorology, evaporation, etc.) and to transmit these details to the model parameters in such a way that it would allow constructing a perfect model which identically reproduces observed variables. A hydrological model remains therefore only a simplification of a real basin and hence cannot perfectly reproduce a real basin response (Beck 1991). This results in deviations between predicted and observed variables. The accuracy of model predictions depends on several factors i.e.: model structure, selected parameter values, external and/or initial model assumptions and others, see further Sect. 4.2.

To reduce such deviations in predictions and to improve the model accuracy, usually three means can be considered: i) improving model structure, ii) adjusting model parameters or iii) collecting more and more accurate data. Improving the model structure usually leads to more complex models through involving additional parameters or input variables which allows better describing the process dynamics within the basin (Blöschl, Montanari 2010). This is not always possible because it usually requires additional data which may not be available. On the contrary, adjusting model parameters is simpler. To this end, a model is calibrated against recorded input-output data in order to determine optimal parameters which give the best output simulation. The model thus better reproduces observed patterns and the simulation accuracy improves. Adjusting model parameters during a calibration process leads, however, to a loss of some of their assumed a priori physical interpretation (Wagener, Gupta 2005). Such inferred parameters should therefore be referred as conceptual parameters or effective parameters (Romanowicz, Beven 2003) rather than physically-based parameters. Finally, collecting more and better data requires longer time and financial investment in measurement campaigns. In this regard, en-

suring high data quality is of higher importance than extending available observations. Better data in terms of quality and information content usually allow for better identification of model parameters. Yet, the improvement in model predictions is tied with the model structure and model ability to reproduce observed patterns (see further Sect. 4.2).

Because it is extremely difficult to obtain satisfying model parameters only from physical basin characteristics, without considering any observed records, calibration is crucial for most models (Wagener, Montanari 2011). Unfortunately, simple models may not satisfactorily imitate basin's processes even after calibration due to excessive simplification (Beck 1991). Alternatively, complex models may require many data for their optimal calibration which often are cost-ineffective compared to the gained improvement in predictions (Montanari et al. 2009). Consequently, a chosen model represents usually a compromise between a very poor model (that behaves incorrect) and a very complex model with a large amount of parameters which cannot be determined due to an identifiability problem (Reichert 2012). Such a problem occurs when available data do not contain enough information to unequivocally identify optimal parameters and an ambiguity in representation and interpretation of past observed patterns remains (Beck 1991).

The lack of calibration data (e.g. streamflow, sediment) and the uncertainty associated with hydrological predictions are therefore seen as major limitations for hydrological science nowadays (Sivapalan 2003; Wagener, Montanari 2011). In the absence of gauged data, hydrologists are forced to search for better tools to make predictions. This means there is a need for models which, on the one hand, are less demanding in calibration data but, on the other hand, better reproduce observed variables (Wagener, Montanari 2011) and consequently are less uncertain (Sivapalan et al. 2003; Montanari 2011; Wagener, Montanari 2011).

2.1.2. Specificity of Small Urbanized Basin (SUB)

Small basins are typically defined as basins with a contributing area up to several dozens of square kilometres (Marshall, Bayliss 1994). This is usually up to 50 km^2 (Ciepielowski, Dąbkowski 2006). Such areas are drained by small local streams which further supply larger rivers.

Due to a small contributing area, on the one hand, small basins react rapidly to rainfall events and the response time usually may be measured in hours (Marshall, Bayliss 1994). Therefore, it is assumed that snow melting and groundwater do not play a significant role in the generation of runoff after a rainfall event. For flood predictions, a basin response may thus be reduced to modelling only the direct surface runoff while omitting baseflow (Banasik et al. 2000). On the other hand, small basins are much more sensitive to local conditions such as land use changes resulting from basin urbanization (WMO 2008; Banasik 2011). In this, a sensitivity of a basin is inversely proportional to its area since small absolute changes lead to relatively large changes in the basin area.

Small urbanized basins (SUB) are characterized as basins in which the ratio of urban sites has been significantly increased over time. This results from a transition from a natural or rural to an urbanized basin, and in small basins usually occurs rapi-

dly (Banasik et at. 2008). This ongoing process occurs until a basin becomes entirely urbanized; the ratio of urban sites equals 100%. Because usually the ratio of urban sites in SUBs is lower than 100%, it is difficult to separate stages prior to and after the urbanization. Typically, prior to the urbanization, a basin possesses a dominating ratio of permeable surfaces (for infiltrating water) such as farmlands or open spaces (e.g. parks, forests, gardens, unpaved roads) with only a little ratio of urban sites, which is typically less than 5% (Marshall, Bayliss 1994). As urbanization goes on, rural areas are consequently substituted by urban sites with impermeable surfaces (e.g. paved streets, parking lots, building roofs), what gradually increases the ratio of urban sites, see Fig. 2.2. for an example.
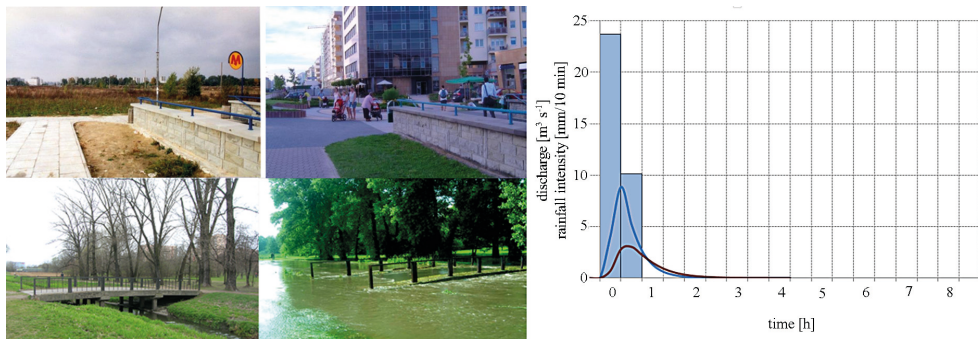
Fig. 2.2. Runoff in a rural and an urbanized basin. Top left panel: transition from a rural (left) to an urbanized (right) basin by an example of Sluzew Creek, Poland; source: ursynow.org.pl. Bottom left panel: example of a small urbanized stream during ordinal flow (left) and high flow (right), Sluzew Creek. Right panel: comparison of runoff in a rural (brown line) and an urbanized (blue) basin in response to the same rainfall (blue histogram), Sluzew Creek; reproduced from Banasik, Ngoc 2010

An increased amount of sealed surfaces reduces permeable surfaces for water infiltration and a possible basin retention and thus strongly modifies a basin hydrology (Hall, Ellis 1985; Byczkowski 1999). Consequently, rainfall water will be quickly drained as a surface runoff. This results in a larger volume of discharged water and faster concentration time (see Fig. 2.2 right panel); time needed for a rainfall water to be discharged into a local stream (Ignar 1993; Christopherson 1997). This is especially visible after heavy rainfalls when the magnitude of discharged runoff increase can be up to few times higher in an urbanized basin in comparison to the state prior to the urbanization, Fig. 2.2 right panel. Thus, during flood conditions small streams can rapidly change into large rivers and may endanger neighbourhood areas (Fig. 2.2 left bottom). Such very short and rapid floods are usually known as flash floods in order to highlight their specific nature (White, Howe 2004). In addition, rainfall floods may be accompanied by associated indirect flooding from combined sewage overflows (CSOs), which occur due to exceeding the drainage capacity (Hall, Ellis 1985).

2.1.3. Difficulties of flood predictions in SUB

While large and economically important basins may have sufficient hydrological gauges for the determination of streamflow, many small to medium-sized catchments are often without any gauging station. In small basins if some observations are available, they are limited to a few years of the most basic hydrological variables such as rainfall and streamflow or water level. These data allow approximating only average yearly values. Often a possibility to continuously or even temporary model river flows is not given (Ciepielowski, Dąbkowski 2006). Unfortunately, in most cases, those regions of the world that suffer most from ungauged network are poor equipped in resources for flood hazard mitigation and adaptation and hence their vulnerability is high (Kapangaziwiri, Hughes 2008).

Such scarce monitoring programs established for small streams is due to several reasons. First, it is difficult to cover all small basins with monitoring even of basic hydrological variables due to a short and rapid basin response during rainfall events. Therefore, traditional observations with one record per day, which proves efficiency in bigger basins, are insufficient in small basins. Hence more frequent observations are required (Ciepielowski, Dąbkowski 2006) and those are expensive, especially when automatic equipment must be used. Second, a regular monitoring program can usually not be set-up in SUB because of a relatively very low flood hazard risk. This means that in case of flooding economical losses and casualties will be relatively small in comparison to larger rivers. Third, to provide a sufficient data set for optimal hydrological model calibration, long time series of observed input-output data are required and such require time to be gathered.

For instance, in Poland from 4 656 rivers only 700 (15%) are gauged (Ciepielowski, Dąbkowski 2006). The monitoring program covers 190 rivers with a watershed area above 500 km^2 and only 1.8% from small rivers with the area less than 50 km^2, whereas 62% (2 919) of all rivers in Poland are defined as small.

Despite the needs, increasing monitoring in reality is limited due to technical, economical and man power limitations and the amount of gauged basins frequently decreases (Kapangaziwiri, Hughes 2008). Therefore, SUBs remain mostly ungauged or poorly gauged (Sivapalan 2003) or become considered as such speaking of current or future land use changes (Sikorska et al. 2012a). Specifically, the term of an ungauged basin refers to a particular basin which fulfills the following condition (EM 1994): »*In the absence of data required for (statistical) parameter estimation for either existing or future conditions, the stream and contributing catchment are declared ungauged*«. This means that, first, a basin which was gauged for particular conditions, e.g. prior to the urbanization, may become ungauged when referring to future changed conditions, e.g. after urbanization (see Sect. 2.1.2). Second, following the international scientific community, International Association of Hydrological Sciences Predictions in Ungauged Basins initiative (PUB initiative), the same basin may be ungauged when speaking of one variable, e.g. sediments, but at the same time gauged in regards to other variables, e.g. water level. The term ungauged refers also to a basin with inadequately observations to enable computation of hydrological variables of interest to the accuracy acceptable for practical applications (Sivapalan et al. 2003).

Lack of input-output observations to calibrate models makes hydrological predictions in SUB extremely difficult (Sect. 2.1.1). Consequently, making predictions without the possibility for a direct model calibration has become a common problem in hydrological practice (Sect. 2.2) and may lead to large uncertainty on the predicted variables (Franks 2002; Sivapalan et al. 2003; Wagener, Gupta 2005), see Sect. 2.3. SUBs are therefore especially interesting for hydrologists. This will not change in the nearest future as long as social (urban) and climate changes continue. The current emphasis of the PUB initiative is to put on improving methods that enable hydrologists to make predictions in basins with limited or no historical observations and on the reduction of the uncertainties associated with these predictions (Sivapalan et al. 2003; Kapangaziwiri, Hughes 2008).

2.2. Coping with predictions in SUB

2.2.1. Conceptual modelling

The general lack of recorded data in SUBs (see Sect. 2.1.3 and 2.2.4) usually prohibits the use of detailed physically-based models with many parameters that have to be inferred from calibration data. Hence conceptual models that require inference of only a few parameters are frequently used to predict the consequences of the future urbanization in SUBs (Sikorska, Banasik 2010; Bocchiola et al. 2011; Sikorska et al. 2012a). Such models link model output and input through the relation with conceptual parameters having a direct interpretation. Hence parameters of conceptual models can be inferred independently from recorded data (Wagener, Montanari 2011) based on catchment indencies via a parametrization process (Kapangaziwiri, Hughes 2008). Thus, conceptual models are important tools in understanding and predicting basin responses to measured or modelled climate and land-use scenarios (McMillan et al. 2010). However, due to a gross simplification of a complex basin system to the form of a conceptual model, they may provide uncertain predictions (Seibert, Beven 2009; Sikorska et al. 2012a).

2.2.2. Parametrization process and its limitation

Parametrization relies on estimating prior information on conceptual model parameters independently from input-output calibration data. Typically, it takes one of the three main approaches (Wagener, Montanari 2011):
• Regionalization;
• Parameter elicitation;
• Parameter transformation from a donor basin.
Apart from the latter, both methods (1) and (2) use commonly available or easily accessible basin attributes such as climate, topography, vegetation, soil properties, annual rainfall, areal potential evapotranspiration, basin area and geology (Chiew, Siriwardena 2005; Boughton, Chiew 2007).
In the regionalization, model parameters are estimated by one of two methods: i) statistical methods or ii) based on regional average values (Kapangaziwiri, Hughes 2008). In regards to i), bivariate or multivariate linear and non-linear regression

relationships are developed between optimized model parameters and some basin attributes for a number of gauged basins. These relationships are next transferred to the basin of interest (e.g. Seibert 1999; Merz, Blöschl 2004; Parajka et al. 2005; Wagener, Wheater 2006; Oudin et al. 2008). In refer to ii), parameter values are mapped to average values for the region, based on the assumption that two or more basins that are located close to each other in real world would have a similar runoff regime. This is justified by smooth changes in climate and basin properties in a physical space (Merz, Blöschl 2004). Parameters are next assigned based on a similarity in soils, rainfall, runoff ratios, etc. between basins.

Model parameters can be also elicited only from local information on physical basin's characteristics such as soil hydraulic properties, meteorology, geology, etc. (e.g., Atkinson et al. 2002; Kapangaziwiri, Hughes 2008).

Finally, model parameter set may be transferred from a basin with similar characteristics (donor basin). A similarity between two basins is assessed based on some measure of hydrological similarity (indicies) e.g. yearly mean streamflow, yearly runoff, precipitation (e.g. McIntyre et al. 2005; Buytaert, Beven 2009; Wagener, Montanari 2011).

All three described approaches allow inferring model parameters for basins with poor data coverage such as SUBs. Such delivered parameters are, however, subject to uncertainty which may lead to significant uncertainty on the predicted variables. This uncertainty has its sources in models structural errors, lack of parameter identifiability during the calibration if conducted, and a lack of reliable relationships between observable basin characteristics and model parameters (e.g. Wagener et al. 2004; Wagener, Wheater 2006; Wagener, Montanari 2011). Moreover, the (1) method allows to transfer only single model parameter values without their mutual relationships (Kapangaziwiri, Hughes 2008). Alternatively, the (3) method allows to include mutual dependencies between parameters. However, model parameters transferred from other basins will include also calibration errors from that basin (Franks 2002). This method additionally needs a careful selection of similar basins that are identified based on some selected group-defined signatures (Nathan, McMahon 1990).

To reduce the uncertainty in predictions, it is still recommended to calibrate conceptual models at least with short-term recorded data (Wagener, Montanari 2011). This allows to better identify model parameters and leads towards a reduction of uncertainty attached to model parameters and consequently to model predictions. As shown by Seibert and Beven (2009), short time series of streamflow observations can greatly help to infer accurate parameter estimates for a conceptual model applied to small and medium-size basins.

2.3. Uncertainty of hydrological predictions in SUB
2.3.1. Why uncertainty? Probabilistic vs. deterministic approach

In hydrological modeling (Sect. 2.1.1), there are two main approaches in use: deterministic and probabilistic. A deterministic model does not involve any randomness into the hydrological process. Therefore, a certain input under certain initial and boundary conditions will always produce the same model output. In contrast,

a probabilistic model (also stochastic) involves randomness into the hydrological process. Thus, despite constant initial or boundary conditions, the process may evolve in several directions (Montanari et al. 2013), see also Fig. 2.3.
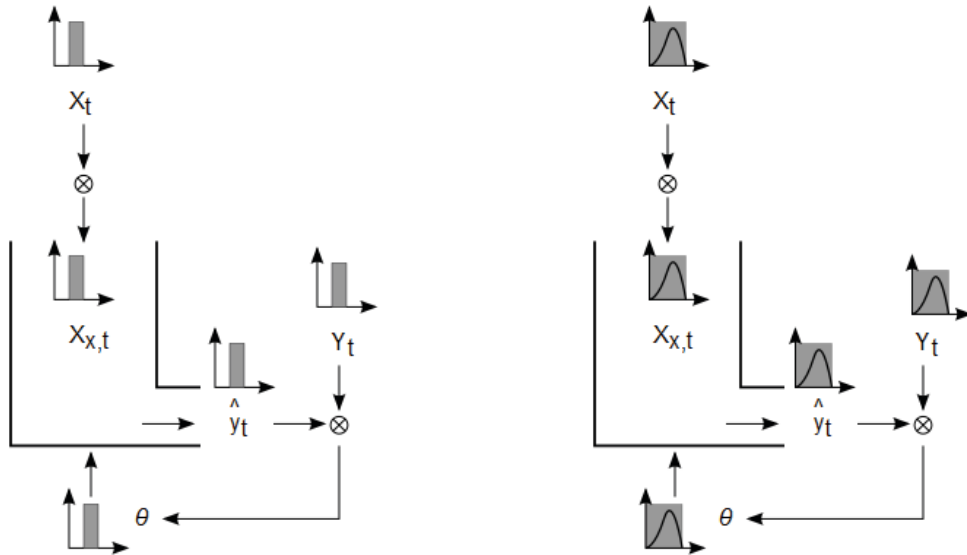
Fig. 2.3. Deterministic (left) vs. probabilistic (right) modelling. Notation:
X – real forcing basin input; X_x – input into the model; y – model output;
Y – real basin output (response); θ – model parameters; t – current time step;
crossed circle – measurements location; based on Wagener et al. 2004.

Hydrological and meteorological events are commonly modelled as random events because it is difficult to define their magnitude, location, time and frequency of occurrence (Ciepielowski, Dąbkowski 2006). Based on their past observations it is possible, however, to deduce on principles of such events by defining the possibility of their occurrence. Unfortunately, due to their high variability, long term observations are usually required for their analysis. Representing model outcomes as random variables allows including prediction uncertainties into model results.

2.3.2. Value of uncertainty analysis in hydrological modelling

Because model outputs are uncertain (see Sect. 2.1.1) uncertainty analysis should be unavoidable in any (hydrological) modeling. Although this is currently emphasized in hydrological community (Sivapalan et al. 2003; Wagener et al. 2004; Efstratiadis, Koutsoyiannis 2010), it is still not a practice to link uncertainties to model predictions and to communicate them to decision makers. Consequently, decisions are often made without the knowledge of their uncertainty or even a possibility of being wrong. This leads to a false society perception of living safe; e.g. in floodplains or behind levees which are usually constructed for a flood with a 1% pro-

bability of occurrence. This means that on average a flood will occur once per 100 years, so relatively rarely. Such a false confidence is illustrated by the example below.

Didactical example

Figure 2.4 presents the importance of uncertainty estimation in flood risk studies. A precise prediction determines here whether the inhabitants of a house are alarmed or not. Two possible predictions are to be considered. I – the house will be flooded – the inhabitants are alarmed; and II – the house will not be flooded – the inhabitants are not alarmed. These two predictions lead to four possible scenarios.

Scenario IA – the house is flooded. Because the inhabitants were alarmed and possibly evacuated, the casualties and economic losses are greatly reduced. Scenario IB – the house is not flooded. The inhabitants were evacuated so the economic losses occur only due to the false alarm; no casualties are borne. Scenario IIA – the house is not flooded. The inhabitants were not evacuated so neither economic losses nor casualties are borne (Fig. 2.4 left). Scenario IIB – the house is flooded. The inhabitants were not evacuated so both economic losses and casualties are expected to be high (Fig. 2.4 right).
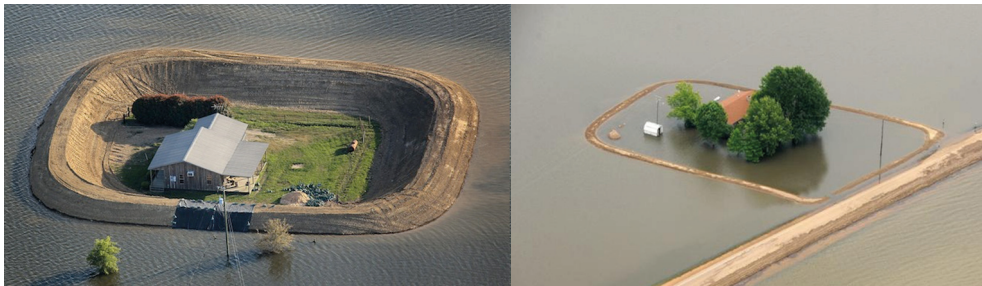
Fig. 2.4. Example of the significance of uncertainty estimation for the house located in floodplains; prediction (II) – the house will not be flooded; scenario A (left) – accurate prediction, the house is not flooded; scenario B (right) – wrong prediction, the house is flooded; source: http://gizmodo.pl/tag/powodz; http://wiadomosci.dziennik.pl

The example above clearly proves the significance of the precise prediction and the knowledge of a possibility of being wrong. If instead of a single prediction II: the house will not be flooded, the uncertainty of this prediction would have been communicated i.e. The house may be flooded, the losses in scenario IIB could have been greatly reduced. Because if a possibility of being flooded was communicated, the prediction II would have leaded to the same actions as in the prediction I; i.e. inhabitant evacuation.

Thus, communicating predictions with uncertainty protects from an apparent belief that derived single estimates are the only true solution and allows keeping alertness (Wagener et al. 2004). The uncertainty consideration can support decision making and usually three main reasons are given (Reichert 2012), namely:
- growing expectations from decision makers for a higher of accuracy and precision in hydrological predictions,

- growing interest in reducing the uncertainty in modelling and a better integration between a model and data,
- increasing knowledge and understanding of hydrological processes within scientists' community.

Uncertainty analysis (UA) allows one to quantify this uncertainty in terms of feasible values that can be pracitally used in decision making process. When properly evaluated, UA provides with estimates which have a statistical interpretation. Providing outcomes with their probability may, however, increase the complexity in decision making because such outcomes should be implemented together with their probability estimates (Rossi et al. 2005).

2.3.3. Uncertainty definition

Despite many hydrological studies on uncertainty, its unique definition is still missing (Wagener et al. 2004) and only a few attempts have been taken to define it. First, by contrast to the determinism, uncertainty may be described as any departure from unachievable ideal of complete determinism (Walker et al. 2003). Second, Funtowicz and Ravetz (1990) described uncertainty as a situation of inadequate information due to inexactness, unreliability, ignorance. Third, uncertainty may be considered, following Montanari (2007), as an additional attribute of information.

Given above, it should become clear that the uncertainty should not only be interpreted as a lack of knowledge that arises from incomplete information or ignorance (Colyvan 2004). Conversely, more information and better knowledge on a certain issue can bring more uncertainty because it allows one to recognize that the analysed process is more complex than it was assumed before. This is illustrated in Fig. 2.5 which presents different levels of uncertainty awareness, starting from an unachievable deterministic approach to an indeterministic approach and a total ignorance at the other side. Between both different levels of the ignorance and uncertainty exist and those include statistical uncertainty, scenario uncertainty and recognized ignorance.

Determinism represents an idealistic situation in which a system and its behaviour are intensively examined and the model perfectly imitates the system. It is worth noting that determinism defined in such a way is not equal to the deterministic approach in modelling, which assumes not that a system is extensively examined and therefore certain but that it can be conceptually described as deterministic, see Sect. 2.2.1. Opposite to the determinism, indeterminacy represents a situation when a mechanism and behaviour of a system are not known precisely (or not at all) and this unawareness (ignorance) cannot be reduced. Statistical uncertainty refers to any

Determinism			Indeterminacy
Statistical uncertainty	Scenario Uncertainty	Recognised Ignorance	Total Ignorance
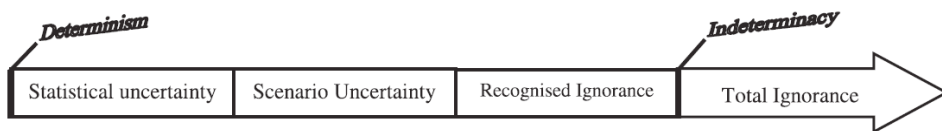
Fig. 2.5 The progressive transition between determinism and total ignorance;
source: Walker et al. 2003

uncertainty that can be described accordingly to statistics. Scenario uncertainty describes the uncertainty in environmental system due to unknown (usually future) conditions and their effects on the system and it cannot be captured by statistical variables. Recognized ignorance represents uncertainty about the mechanisms and functional relationships of the system. Finally, a total ignorance describes a situation when ignorance is not yet recognized due to the lack of modeller's knowledge. This ignorance cannot be reduced unless it becomes aware or investigated (e.g. due to research) and thus will develop into recognize ignorance.

In hydrological modeling, a notion 'uncertainty' alone is, however, uninformative and should always be followed by an additional notation to what certain object it refers to, e.g. parameters uncertainty, uncertainty of predictions (flooding) (Montanari 2007). Mathematically, the uncertainty of an event (model output) can be expressed by a probability of this event occurrence (Box, Tiao 1992; Winkler 1996; Reichert 2011).

2.3.4. Uncertainty of hydrological model predictions (predictive uncertainty)

A hydrological model cannot perfectly reproduce the process that it models (Sect. 2.1.1) and thus its output is uncertain. In hydrological community, this uncertainty is called predictive uncertainty (elsewhere model outcome uncertainty or prediction uncertainty). An occurrence of this uncertainty demonstrates a discrepancy between an observed output and predicted model output, which is called a prediction error. If an observed output is available, it may be compared with predictables in order to estimate this error, which allows one for estimating model credibility.

Conceptually, there is only one predictive uncertainty which refers to model output and sometimes is called total predictive uncertainty (elsewhere integral predictive uncertainty) (Winkler 1996). However, it is commonly agreed that the predictive uncertainty has different sources (Walker et al. 2003; Wagener et al. 2004). For hydrological studies, it can be important to locate sources of uncertainty and assess their contributions. This information may support modelling by pinpointing the weakest part in hydrological modelling. Thus, a distinction between uncertainty due to various contributing sources has practical aspects. To this end, the predictive uncertainty can be decomposed into various contributors (uncertainty sources) what can be, however, done only under certain assumptions. The most common assumption relies on source additives.

2.3.5. Sources of the predictive uncertainty

The predictive uncertainty sources in hydrological modelling are typically represented by (Walker et al. 2003; Wagener et al. 2004):
1. Model structure deficits (Ajami et al. 2007; Reichert, Mieleitner 2009; Renard et al. 2011; Honti et al. 2013) see Sect. 3.2.2, which arise with mapping the real system to a mathematical model structure. This, in particular, includes relationships and functions between all model elements as inputs, outputs and variables, initial boundaries, functional forms, parameters, equations, assumptions and mathematical algorithms (Walker et al. 2003).

2. Parameter uncertainties (Beven, Binley 1992; Ajami et al. 2007; Vrugt et al. 2008a) see Sect. 3.2.3. In particular, the uncertainty in model parameters is related to the type of parameters. Namely, exact parameters such as universal constants (e.g. $\pi = 3.141...$) and fixed parameters, which have been extensively investigated previously (e.g. earth gravity) are assumed to be certain. All others should be considered as uncertain.
3. Input uncertainty (Kavetski et al. 2002, 2006a, b; Renard et al. 2010; Vrugt et al. 2008b; McMillan et al. 2011), see Sect. 3.2.4, that is associated with input variable that forces model behaviour (external driving force). Additional errors may also occur during measuring process of input data or data pre-processing.
4. Output uncertainty or uncertainty in calibration data which occurs due to uncertainty in output data for model calibration and is mostly caused by observational errors (measurement) (Schmidt 2002; Di Baldassarre, Montanari 2009; McMillan et al. 2010).

Traditionally in hydrological modelling, priority has been given to model structure errors and model parameter uncertainty (Vrugt et al. 2008a). Other sources have been usually assumed not to be of the considerable importance due to small influence on the model's output (input uncertainty) or a common belief in a high accuracy of calibration data (output uncertainty). However, even if not all of those sources have to be significant in every model and every basin, all should be properly acknowledged.

Unfortunately, while making predictions in SUB, it is not always possible to distinguish between diverse uncertainty sources because of a noticeable dependency between all of them, especially between a model structure and inferred parameter uncertainties (Walker et al. 2003).

2.3.6. Predictive uncertainty nature

The acknowledged predictive uncertainty is subjected to epistemic and aleatoric uncertainty and thus may be reducible or not (Aronica et al. 2013). The predictive uncertainty may be reduced (epistemic uncertainty) if it occurs due to imperfect knowledge on the system, limited or inaccurate data, measurements error, limited understanding of the system, imperfect models, subjective judgment. Alternatively, the predictive uncertainty cannot be reduced (aleatoric uncertainty else variability uncertainty) if it is emerging from the variability of the complex system being described. Hence it is caused by variation in external input data, input functions, parameters and model structure due to randomness in an environment itself, incoherence in human behaviour, social, cultural and economic dynamics and technological variability.

It must be stressed here that the predictive uncertainty usually comprises both types i.e. epistemic and aleatoric uncertainties. Thus, usually only a part of it (epistemic uncertainty) may be reduced while some uncertainty (aleatoric uncertainty) always remains. In this, identification of the predictive uncertainty nature (reducible or irreducible) is crucial when assessing whether more research (empirical efforts) would bring more information and significantly reduce the current uncertainty or rather other model approaches should be chosen (Winkler 1996). The uncertainty analysis allows locating significant uncertainty sources and in this way supports making decisions.

3. METHODS – BAYESIAN UNCERTAINTY ANALYSIS IN SUB
3.1. Preface to methods chapter

3.1.1. Stochastic description of the rainfall-runoff process

A rainfall-runoff (RR) process within a basin may be characterized by the real precipitation, evaporation, etc. that induces the real basin response known as real runoff (streamflow, water level or sediment concentration), see Fig. 3.1. In a similar fashion, an RR model imitates the real system by transforming an input variable (e.g. rainfall) into an output variable (e.g. runoff). Unfortunately, due to measurement and perception errors (Sect. 2.4.4 and 3.2) real variables are very difficult to measure in practice and instead observed variables are measured. This is represented by the equation of observed output and input:

$$Y_o = Y + e_y \text{ and } X_o = X + e_x \qquad (3.1)$$

Where, Y_o, Y, e_y are observed and real output and observational error, X_o, X and e_x are observed and real input and its error.

Also, modelled output usually differs from the observed output due to structural limitations of the model (Sect. 3.2). A calibration of a model allows reducing errors of a mismatch in predictions but some errors usually remain due to many unknowns involved in hydrological modelling (Reichert, Schuwirth 2012). Additional errors may occur when transferring observed forcing variable (e.g. punctual precipitation) into input required by model (e.g. areal precipitation). This is represented on Fig. 3.1.

Note that errors integrate along arrows so that the modelled outcome y (on Fig. 3.1.) based on the observed input variable (X_o) contains errors due to: i) measurement and transformation errors of the observed input variable ($X \rightarrow X_o \rightarrow X_x$), ii) structural errors of the model ($X_x \rightarrow y$) iii) errors of model parameters (θ) and iv) measurement errors of the observed modelled output ($Y \rightarrow Y_o$) if the model is calibrated. All those errors contribute to the predictive uncertainty of the variable predicted by a RR model (e.g. y). However, the importance of diverse errors may be different and thus errors may contribute unequally to the predictive uncertainty.

3.1.2. Formulation of the stochastic rainfall-runoff model

A deterministic hydrological model M that transforms input data (X) into modelled output y can be represented by a function of model parameters θ^M and X (Kavetski et al. 2006a, b) as: $y = M(X, \theta^M)$, see Eq. 2.1. A bold font indicates a vector. A model M aims at imitating the real system variable Y. When this imitation is imperfect (Sect. 3.2.2), additional errors arise due to i) measurement noise of the system variable represented by ε and ii) errors in input and structural limitations of the hydrological model. These last two error sources produce what is named model bias represented by B. A statistical description of the bias is not a trivial task and requires undertaking research of its own (Del Giudice et al. 2013). Recently, the inclusion of these errors into predictions has been proposed, by combining a deterministic model M with both error terms (Reichert, Schuwirth 2012):
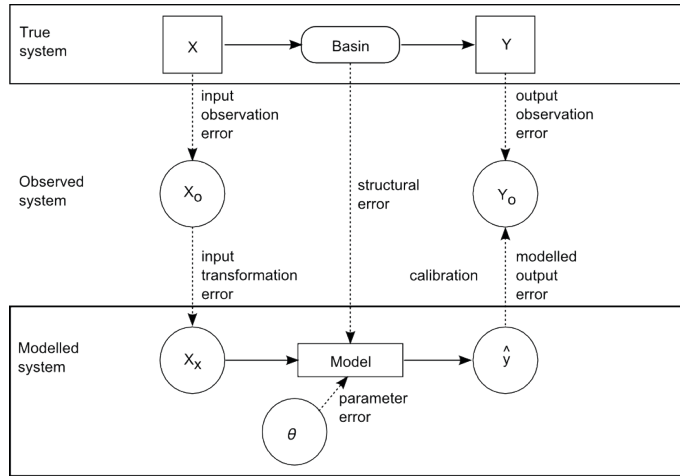
Fig. 3.1. Schema of possible uncertainty sources in hydrological modelling. Dashed lines pinpoint location of possible errors. Random quantities are shown in ellipsoids, deterministic in squares. Notation: X, Y represent a forcing variable and model response that occurred in the reality; X_o and Y_o represent observed variables respectively; X_x is transformed observed input variable into the model; y is modelled output; bold font indicates vectors.

$$Y = y(X, \theta^M) + B(X, \theta^B) + \varepsilon(\theta^\varepsilon) \tag{3.2}$$

Where Y is the predicted stochastic output of M when accounting for errors. θ^ε and θ^B represent parameters of the measurement error and of the bias, respectively. $B(X, \theta^B)$ represents a stochastic process and cannot be known in advance. ε could be only estimated if the Y is known. Thus, if only insufficient information on both errors is available, it is a common practise to model $B(X, \theta^B) + \varepsilon(\theta^\varepsilon)$ as a single error term E:

$$Y = y(X, \theta^M) + E(\theta^E) \tag{3.3}$$

Where $\theta^E = \{\theta^B; \theta^\varepsilon\}$ and θ^M represents a vector with all model parameters: $\theta^M = \{\theta^{M1}, \theta^{M2},...,\theta^{Mn}\}$.

3.2. Sources of predictive uncertainty

The predictive uncertainty (PU) of model predictions is modelled as an accumulated uncertainty of four mentioned uncertainty sources (see Fig. 3.2 and Sect. 2.3.4). These sources are subject to uncertainty of diverse origin.

3.2.1. Uncertainty of model structure

Uncertainty in a model structure is unavoidable in hydrological modelling whenever an imperfect model is constructed (Wagener, Montanari 2011). In general, this structural uncertainty may be caused by (Reichert 2011):

- inadequate selection of model variables and processes,
- inadequate selection of process formulations,
- inadequate choice of the spatial and temporal resolution of the model.

Consequently, a model, due to its structural limitations, will reproduce an observed variable with an prediction error (Gupta et al. 1998). Unfortunately, a formal description of this error is difficult. Indeed, it is extremely difficult to describe model errors in the form of relations; otherwise, the model structure could have been improved, e.g. by including a correcting factor (Reichert, Schuwirth 2012). To reduce errors in predictions, a model is usually calibrated against observed data. This allows adjusting model parameters so that the predicted variable can better reproduce the observed variable.

A calibration of the model, however, has two difficulties. First, if the model structure remains untouched, the model calibrated to some observed data will most likely not be able to predict outcomes for new data sets with the same accuracy. Second, some models, even after calibration, cannot reproduce data in a satisfying way (Sect. 2.1.1). This is mainly caused by: i) too far going simplifications of a model structure, ii) input uncertainty in the forcing variable and/or iii) a poor data set which does not contain enough information to infer all model parameters which results in model overparametrization. This problem may be of a particular concern for SUBs that are usually poorly gauged (Sect. 2.1.3) and for which typically simple conceptual models must be applied (Sikorska et al. 2012a).

3.2.2. Uncertainty in model parameters

Parameters of hydrological models are usually estimated from observed output data or physical properties of a basin and typically consist of a basin area, a runoff coefficient, an impervious area and other basin characteristics. Traditionally, parameter values or ranges have to be chosen prior to the calibration. Such an arbitrary choice of parameters is subject to the uncertainty. As stated in Sect. 2.3.4, only exact and fixed parameters may be assumed as constant i.e. without any uncertainty. All others should be considered as uncertain.

Parameter uncertainty is strongly related to the model structure uncertainty (Sect. 3.2.2). In the case of no bias being apparent in the model, parameter uncertainty would tend to zero as the quantity of calibration data approaches infinity.

Typically, the parameter uncertainty is of the highest concern in a hydrological community. This is due to a common (mistaken) belief that model parameters are the most unsure component in modelling since they need calibration to predict a variable in a satisfying way (Sect. 2.1.1). Conceptually, model parameters are also the easiest component of uncertainty to account for because of their explicit description.

3.2.3. Model's input uncertainty

Input uncertainty describes the uncertainty in the observed variable that drives the model (see Sect. 2.3.4). Traditionally in hydrological modelling, input uncertainty was assumed to be insignificant relatively to other sources and therefore has been frequently disregarded by hydrologists. However, it has been recognized that

a model input as a forcing variable has a crucial influence on model predictions and consequently on model accuracy (Kavetski et al. 2002; Kuczera et al. 2006).

The input into rainfall-runoff (RR) models usually consists of observed precipitation and sometimes evaporation. The uncertainty of evaporation data arises mostly from the measurement errors and may be considered as 2-5% of measured values (WMO 2008). In SUBs, due to smooth and slow changes in evaporation over the spatial area and a small contributing area, the evaporation over the entire basin area can be assumed to be constant. The error due to the spatial variability in evaporation is thus less significant.

In contrast to this, precipitation is characterized by a significant spatial and temporal variability of rainfall fields over the basin area (Fig. 3.2). Ideally, input precipitation into RR models should represent this variation. An areal measure of precipitation can be achieved e.g. with radar data. However, their high costs and poor spatial resolution (few kilometers) usually limit practical applications in SUBs. Instead, precipitation within a basin is traditionally measured with point rain gauges unevenly spread over an area of interest. The advantage is a relatively low cost of maintenance and data-gathering. Unfortunately, such irregular rain gauge networks cannot capture rainfall field variability and are thus limited to measure only a punctual rainfall occurrence (Kavetski et al. 2002, 2006; Bárdossy, Das 2008; Moulin et al. 2009; McMillan et al. 2011). Because the RR model requires areal precipitation as an input, measured punctual rainfall must be averaged over the basin area.

Given the aforementioned considerations, even if errors in precipitation measures alone can be assumed to be small and represent only 3-7% of measured values [WMO 2008], the uncertainty in input rainfall to RR models must still be considered. In particular, this may be due to, (McMillan et al. 2011):

•   Usually poor representation of rainfall fields over the entire basin by a (small) set of punctual gauges which is the case for most of the SUBs. It is also not uncommon for a single gauge to be located close enough to be used.

Fig. 3.2. Spatial variation of a rainfall field over a city, source: http://forum.xcitefun.net

- Interpolation of rainfalls between measures observed on rain gauges, which is mandatory due to a sparse rainfall gauge network. Although this may not play a significant role in small rural basins, SUBs are particularly subjected to such errors as a result of i) local rainfalls of which coverage is limited only to some districts within the city and ii) cities' influences on clouds (warmness, wind, etc.).
- Measurement error in commonly used tipping bucket rain gauges. This includes both systematic and random errors, as well as those due to local influencing factors (mechanical limitations) such as wind effects, evaporation losses, influence of neighbourhood (trees, buildings, etc.).

Consequently, input precipitation into RR models extracted from point rain gauges is expected to be highly uncertain.

3.2.4. Uncertainty in calibration data

Measurement uncertainty of the modelled quantity is called uncertainty in calibration data or output uncertainty. The accuracy in calibration data determines the reliability of hydrological predictions and is therefore of particular importance (Domeneghetti et al. 2012). The uncertainty in calibration data occurs due to the fact that the real value of a variable can never be precisely captured during measurements (Walker et al. 2003), see also Fig. 3.2. Therefore, measured data are subject to errors which may stem from:
- sampling,
- inaccuracy/imprecision in measurements,
- transformation errors when mapping directly measured variables into desired variables (e.g. water levels to streamflow by a rating curve).

Uncertainty in calibration data and especially its influence on predictive uncertainty is rarely assessed quantitatively by hydrologists, for two main reasons (Sikorska et al. 2013). Firstly, it is difficult to make any statement on output uncertainty if data are transformed from other quantities. Secondly, modellers often work only with the derived quantities and not with the raw data.

Given these facts, it is a common practice to assume that this uncertainty is much smaller than that from model parameters, model structure or input, and is thus assumed to be negligibly small (Di Baldassarre, Montanari 2009; Di Baldassarre, Claps 2011; Sikorska et al. 2013). This can be true when special efforts are put into maintenance procedures and better equipment which allow for a significant reduction of the output uncertainty. In all other situations, however, this uncertainty should be considered cautiously. This is especially so as some uncertainty always remains despite sufficient maintenance (see Sect. 2.3.4).

Calibration data typically refer to streamflows for rainfall-runoff (RR). Hence, the uncertainty of this variable is further discussed.

3.2.5. Uncertainty of streamflow data

The uncertainty of streamflow data is strongly dictated by the measurement method. If streamflow measurements are of a high quality, e.g. gather in good con-

ditions, their measurement errors are assumed to be rather small (5%) (WMO 2008). Such small errors can be achieved when streamflows are gathered by current meters with the commonly used area-velocity method (Le Coz 2012). This method links streamflow to a cross sectional area and an average velocity which can be measured with the average error of 2-5% (Di Baldassarre, Montanari 2009; WMO 2008). Although very convenient, this method becomes impracticable in field conditions when continuous or frequent data are required due to time consuming measurements required to obtain a single stream-flow record.

RR models require, however, continuous streamflow data for their calibration. Thus, streamflows are usually computed from easier to continuously measured water levels with the use of a hydraulic model which relates streamflows to water levels, a water level-runoff (LR) model (Sikorska et al. 2013). A LR is usually represented by a rating curve (RC) which consists of an empirical relationship and therefore must be calibrated for a certain cross section on data obtained from hydrometric measurements (Le Coz 2012) Alternatively to an RC, a numerical hydraulic model can be constructed. This is, however, less practical because more data are required, e.g. detailed data on the river channel properties which are more often than not unavailable for SUBs (Sikorska et al. 2013). A measurement error of water levels can be assumed to be small, in the range of 1-2 cm (WMO 2008).

Unfortunately, using RCs to infer streamflow records is not free from error (Di Baldassarre et al. 2012; Sikorska et al. 2013), which is mostly caused by:
- Uncertainty in measured data used to calibrate an RC (punctual records of streamflow-water level relations);
- Structural and physical limitations of the RC method due to assumptions of steady flow conditions, neglecting hysteresis effect, or simplifying a cross section structure to a manageable shape;
- Uncertainty in method's parameters due to temporal and seasonal changes of hydrological conditions within a certain cross section; as seasonal variation of vegetation, temporal movements of a stream bed, variation of a cross section shape, etc.;
- Extrapolation of an RC beyond the measured (or recommended) range.

It has been shown that the latter errors dominate among all other sources of uncertainty in RCs (Domeneghetti et al. 2012). Unfortunately, calibration data for RC are often limited only to normal conditions when the interest lies in flood flows (Pappenberger et al. 2006). Thus, in flooding studies, it is usually a necessity to extrapolate an RC in order to obtain streamflows for RR model calibration.

All these factors contribute to overall RC uncertainty which may be even up to 25% in the extrapolation range (Kuczera 1996; Di Baldassarre, Montanari 2009; Di Baldassarre et al. 2012). This quantitative contribution is, however, not generalizable since it is strongly related to a case study, available data and individual cross section characteristics. For instance, a bed movement and seasonal changes will not be observed in SUBs with an artificial channel. Interestingly, it is an often neglected fact that the uncertainty in the RC propagates through the RC method and is further linked to streamflow records computed with the RC. This uncertainty will consequently influence RR model predictions.

3.3. Introduction to the Bayesian inference

The uncertainty of predictions as represented in the Sect. 3.2 are only meaningful if they have a clearly defined interpretation. Unfortunately, many uncertainty approaches that are commonly in use do not provide uncertainties that can be statistically interpreted. Among others, the most popular approach in hydrology is the generalized likelihood uncertainty estimation or shortly GLUE (Beven, Binley 1992; Romanowicz, Beven 2003; Montanari 2007). This technique relies on a subjective likelihood measure which weighs the probabilities associated with different parameter sets in order to derive the posterior distribution of output variable. Thus, every possible model outcome arrived from the defined parameter space is weighed with this likelihood measure (Romanowicz, Beven 2006). The key feature of GLUE is that the likelihood measure is specified as an objective function. Thus, the estimated uncertainty depends largely on the subjectively specified likelihood measure (objective function) (McIntyre et al. 2002). This likelihood measure should not be interpreted as a statistical likelihood estimator unless it is explicitly specified as such (e.g. Romanowicz et al. 1994). The form of GLUE with statistically described likelihood is refereed sometimes as a formal GLUE (Romanowicz, Beven 2006).

The principle of GLUE lies in mapping all uncertainty in prediction entirely to the parameter uncertainty and in propagating this uncertainty through the model structure in order to estimate uncertainty of a modelled output. Because conceptually it is easy to compute, GLUE is a common choice to estimate uncertainty in hydrological studies. The main drawback of the classical GLUE, i.e. without statistically described likelihood, is the assumption on model correctness. As a result of this, model parameters compensate for the model error. If the model does not reproduce perfectly observed data, this assumption will lead to the increase in parameter uncertainty. However, if many data points are available, the parameter uncertainty becomes low and the associated uncertainties usually are underestimated. Also, model parameters mapped with error of diverse sources become hardly interpretable. Finally, derived uncertainty of predictions does not have necessarily a probabilistic interpretation and may not reflect the real situation. Thus, such estimated uncertainty becomes unhelpful for further studies and applications, and concern a question of their implementation into decision making processes.

In contrast to that, a formal Bayesian approach is based on a subjective interpretation of probabilities and is fully consistent with probability calculus. The principle of the Bayesian approach lies in the Bayes' theorem.

3.3.1. Bayes' theorem and Bayesian probability

The Bayes' theorem, first introduced by Thomas Bayes (1702-1761) and further developed to the nowadays form by Pierre-Simon Laplace (1749-1827), uses the evidence that an event has occurred in the past to calculate the probability that it will occur in the future. Following probability theory, it is expressed as the conditional probability of an event given the probability of another event which has already occurred (Box, Tiao 1992; Gillies 2000). An example of conditional probability application in hydrology is given below.

Didactical example

Consider a following situation, taken from Reddy (1997), a study of daily rainfalls at a certain rain gauge station has revealed that in July the probability of a rainy day following a rainy day at this certain rain gauge is 0.5. If it is observed that a certain day in July is a rainy day, what is the probability that the next two days will also be rainy?

Following some initial rainy day A_0, let A_1 be the first rainy day. A_2 will describe the second rainy day. The probability of A_1 being the rainy day is, from the given information, equal to 0.5 because A_0 has already occurred. Then the probability that the second day A_2 is also a rainy day given that the first day A_1 is a rainy day is now sought for. This is $P(A_2|A_1)$:

$$P(A_2|A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} \quad \text{or} \quad P(A_1 \cap A_2) = P(A_1)\, P(A_2|A_1) \qquad (3.4)$$

$P(A_2|A_1)$ describes now the probability that the following day is a rainy day if the day before is a rainy day. From the given information, this probability also equals 0.5. Thus, the probability that both A_1 and A_2 are rainy days will be described by $P(A_1 \cap A_2)$:

$$P(A_1 \cap A_2) = P(A_1)\, P(A_2|A_1) = 0.5 \times 0.5 = 0.25 \qquad (3.5)$$

If A_1 and A_2 were independent events, they occurred independently from each other, the occurrence of A_2 was not affected by the occurrence of A_1.

Thus $P(A_2|A_1) = P(A_2)$. In the example above this could be considered if A_1 and A_2 are not sequential days.

Bayesian probability

A probability in the Bayesian framework is interpreted as a degree of belief and consequently probabilities are to some degree subjective. This subjectiveness is expressed by a combination of one's belief and the evidence i.e. proved by data [Gillies 2000]. Hence the Bayes' theorem links the degree of a belief in a proposition before and after accounting for the evidence. The application of the Bayes' theorem to update beliefs is called Bayesian inference.

Since the Bayes's theorem considers a subjective interpretation, the resulting probability will alter depending on the state of a belief (subjective) and access to the evidence (data) and may lead do different results when evaluating by different persons (see the example below).

Didactical example

Consider a following coin-tossing experiment, taken from Sivia [1996]; one is tossing a coin n times. By fair, one would expect to observe heads (or tails) in 50% of all n flips assuming that the coin-tosser does not control the initial conditions of the flip e.g. angular conditions. To express the belief of a fair coin, let denote the bias-weighting by H. Thus, $H = 1$ and $H = 0$ can represent a coin which produces always a head, a double-headed coin, and a tail, a double-tailed coin, respectively.

H = 0.5 would represent a fair coin, headed-tailed. A person's belief will represent now the state of knowledge about the coin fairness; how one believes these different states of H to be true. This belief will alter in the light of data when the number of executed tosses n is increasing. This inference about the coin fairness is summarized by the conditional probability: $\text{prob}(H|\{data\}, I)$. I represents the conditioning information as initial information of the coin or the experiment; e.g. that the tosses are independent.

As shown by the example, if only a few data are available, the prior strongly influences the current belief about the coin fairness. This effect vanishes with more data becoming available. The same effect can be observed in regard to the choice of the alternative prior distributions. Different priors have a strong effect on the current belief if only a few data are examined. If numerous data are available, different priors should lead to the same updated belief.

The Bayesian subjective interpretation of the probability is in contrast to the classical (relative-frequency) approach, where probabilities describe limited relative frequencies and therefore are assumed as being objective. Thus, many scientists and statisticians feel uncomfortable with the subjective Bayesian approach (Lele, Allen 2006). In real applications, however, a real objectiveness is almost never obtainable due to many (subjective) assumptions which have to be made (Winkler 1996; Gelman et al. 2003). Consequently, the objective probability would remain only idealogical (but never reached) and the subjective interpretation of a probability would turn into realistic.

3.3.2. Bayesian approach principle

Bayesian concept
One of the strengths of the Bayesian approach is its ease to derive the predictive distribution. Accordingly to the Bayes' theorem, the knowledge about model parameters θ^M is represented as a random variable. The probabilistic assumption on θ^M is expressed by a distribution that describes a subjective belief about their values $p(\theta^M)$ which is called as prior distribution or prior. The 'prior' stresses here that a belief is constructed before considering any evidence of data. The information contained in (calibration) data (Y^C) may, however, alter the current belief leading to stating the new updated belief called as posterior or posterior distribution, $p(\theta^M|Y^C)$. In the same fashion, any environmental system or model can be also represented by a probability distribution based on the belief of its behaviour. This system behaviour can be further described by the conditional belief about system observations given the parameter values called as likelihood function $p(Y^C|\theta^M)$.

Framing belief on model parameters
To be applied, Bayesian statistics requires a quantitative formulation of the current knowledge. This can be done conceptually by relative frequencies within the probability theory (Reichert 2012). Relative frequencies are expressed as a proportion of all given values in an interval and therefore allow including different degrees of belief.

The prior remains constant under constant circumstances but strongly depends on the familiarity and possessed knowledge of the person which constructs its own

belief about the system and therefore may be incomplete. To minimize such sub-jectiveness, the prior represents usually beliefs of a scientists group from the filed of the interest, experts, instead of a single person's belief. Thus, it reflects a current state of the scientific community knowledge on a certain issue. Experts are here di-stinguished from scientists who conduct and manage the entire uncertainty analysis process (e.g. model predictions). Instead, experts are not necessary directly involved in the uncertainty analysis but their knowledge may be used on different steps of this analysis as for instance to elicit the prior. Using experts' knowledge leads to results that become representative in term of the current state of scientific knowledge.

Unfortunately, eliciting a prior distribution may be difficult in hydrology becau-se for many scientists or experts it is challenging to express precisely their knowledge in a probability fashion (Garthwaite et al. 2005). Most of all, because it is difficult to visualize parameters of a deterministic hydrological model as continuous probability distributions. Also, eliciting a full distribution with which an expert is totally comfor-table may pose problems for many experts (West 1988). Instead frequencies pose less problems (Lele, Allen 2006) and therefore in many cases the prior is first inferred as frequencies and after that transformed to probabilities (Sikorska et al. 2012b).

Learning from the data – Bayesian updating

The principle of the Bayesian inference is to use data to update prior infor-mation on model parameters (Eq. 3.6). The Bayesian updating may be therefore interpreted as a learning process. That is, a transformation from the established prior $p(\theta^M)$ to the posterior $p(\theta^M|Y^C)$. Formally, it reflects what have been learned about the assumed a'priori model parameters $p(\theta^M)$ from a consideration of the calibration data Y^C (Gelman et al. 2003):

$$p(\theta^M|Y^C) = \frac{p(\theta^M)\, p\,(Y^C|\theta^M)}{p(Y^C)} \tag{3.6}$$

The goodness of the learning process depends on assumptions about model errors. Typically, a posterior becomes narrower during the learning process. A poste-rior may become wider if there is an evidence for that in calibration data.

3.3.3. Advantages and disadvantages of Bayesian approach

Bayesian statistics has been shown to be conceptually more satisfying than other approaches of uncertainty analysis in hydrological studies (Mantovan, Todini 2006; Vrugt et al. 2008a; Yang et al. 2008; Sikorska et al. 2012a). This is namely due to following (Gelfand, Smith 1990):
• It provides a natural and constant principled way of combining prior informa-tion with data. The current knowledge about the model is summarized by the prior distribution that is used for future analysis. This prior can be formulated i) based on previous studies or short data series and/or local information, ii) elicited from the experts' knowledge without any field data. It may be next incorporated with available observations (data) leading to the updated know-ledge – posterior. If new data become available, the previous posterior becomes

the prior for the next evaluation and may be updated to the new posterior. All inferences logically follow the Bayes' theorem (see Sect. 3.3.1).
- It provides inferences that are conditional on the data and inferred probability distributions that are exact.
- It provides interpretable probability distributions with a real probabilistic interpretation because all analysis follows the basic probability axioms. Thus, derived uncertainty bands represent truly probabilities and derived parameter distributions reflect the real parameter densities.
- Derived posterior distributions have to lie within the support of the prior (Eq. 3.5). This may be particularly useful for inferring nonnegative parameters by setting the probability of negative values to zero.
- Bayesian statistics requires an explicit formulation of the error process. This error has an interpretation because of transparent assumptions.
- It gives the possibility to separate the sources of uncertainty and following that, by assessing their relevance, it allows one to pinpoint where additional steps should be taken to reduce the uncertainty and to improve model predictions (when possible).
- It allows for models comparison and thus for a model selection (not considered in this thesis).
- It allows assessing benefits of the investigation ahead of it; for instance if a new field experiment can be used to limit the uncertainties in predictions or rather another model should be applied.
- It does not necessary require observed output data to provide with probabilistic predictions since current prior information already allows to make predictions without the need for prior updating. Therefore, if a model, prior and a likelihood have been established, the prior parameter distribution can be sampled and corresponding ensemble of predictions can be already given (Wagener, Montanari 2011).

Because of the reasons given above, the Bayesian approach can be also applied in basins without long term observations as in SUBs or in generally poorly gauged basins. For examples of its application see e.g. Beck and Katafygiotis (1998), Sivia and Skilling (2006), Wagener and Montanari (2011), Zhang et al. (2011) or Sikorska et al. (2012a).

Despite many advantages, Bayesian analysis presents some limitations (Gelfand, Smith 1990):
- It requires a formulation of the likelihood which may be difficult to formulate.
- There is no unique approved way to elicit a prior. Thus, formulating current knowledge in term of probability may be problematic (Scholten et al. 2013).
- Computations may become problematic and costly in time if hydrological model is slow in evaluations because many model runs must be performed to find the posterior.
- Numerical problems may occur when sampling from posterior distributions for obtaining simulations because each time slightly different realizations are obtained. As a result of the Bayesian inference probability distributions are derived which are exact. To approximate the predictive distributions of modelled output, model parameters usually must be sampled from these distributions. These random parameter samples are next used to compute corresponding model realisations.

3.4. Bayesian inference

3.4.1. Bayesian learning − calibration of a model

Model predictions

A model response is represented by a probability density function pdf. Predictions of the model M are then formulated as a predictive distribution $p(Y)$ that is calculated by marginalizing the joint distribution of the parameters and Y (Congdon 2003).

$$p(Y) = \int p(Y|\theta^M)p(\theta^M)d\theta^M \tag{3.7}$$

Where $p(Y|\theta^M)$ is the likelihood function which is proportional to the probability that the observations could have been derived by the parameter set θ^M_i (Congdon 2003). Thus, the likelihood assesses the probability of observing data arising from the assumed hypothesis which is represented by candidate model parameters θ^M_i.

Bayesian learning

A prior belief on model parameters $p(\theta^M)$ is formulated without accounting for any evidence in calibration data. This evidence in a hydrological model M is represented by the observed data (output) Y^C:

$$Y^C = \{Y^1, Y^2,..., Y^n\} \tag{3.8}$$

The principle of Bayesian learning from data Y^C (also Bayesian inference), described in Sect. 3.3.2, is to improve current beliefs (knowledge) represented by $p(\theta^M)$ to the new (updated) current beliefs which now become posterior beliefs or in terms of probability – posterior distribution of parameters. This posterior is a combination of the prior knowledge and the data and the likelihood function. This posterior is represented as a conditional probability of obtaining θ^M given the data Y^C so as $p(\theta^M|Y^C)$. According to the Eqs. 3.6 and 3.7, the belief on θ^M becomes (Congdon 2003):

$$p(\theta^M|Y^C) = \frac{p(\theta^M)\,p(Y^C|\theta^M)}{p(Y^C)} = \frac{p(\theta^M)\,p(Y^C|\theta^M)}{\int p(Y^C|\theta^M)\,p(\theta^M)d\theta^M} \tag{3.9}$$

Because $\int p(Y^C|\theta^M)\,p(\theta^M)d\theta^M$ is difficult to evaluate, usually the proportional relationship is sufficient for the probability approximation:

$$p(\theta^M|Y^C) \propto p(\theta^M)\,p(Y^C|\theta^M) \tag{3.10}$$

Accordingly to the Bayes' theorem (Eq. 3.6), the posterior distribution of the model predictions in Eq. 3.7 will become now, after having observed data (Gelman et al. 1996):
Where $p(\theta^M|Y^C)$ is a posterior distribution of parameters.

$$p(Y|Y^C) = \int p(Y^C|\theta^M)\,p(\theta^M|Y^C)\,d\theta^M \tag{3.11}$$

3.4.2. Posterior analysis

Marginal distributions

Comparing the estimated posterior distribution of model parameters $p(\theta^M|Y^C)$ with the prior $p(\theta^M)$ gives an estimate of what have been learnt form the calibration data (Y^C) (see Sect. 3.3.1). The prior is usually expressed as standard pdf for which characteristic values such as a mean or a standard deviation can be derived and therefore the prior may be described in terms of mathematical equations. The posterior, however, usually does not result in a standard distribution and thus cannot be captured in the form of simple mathematical equations. Therefore, a comparison of prior and posterior can be practically assessed by graphical analysis while both pdfs are plotted together (Reichert 2011), see the example in Fig. 3.3.
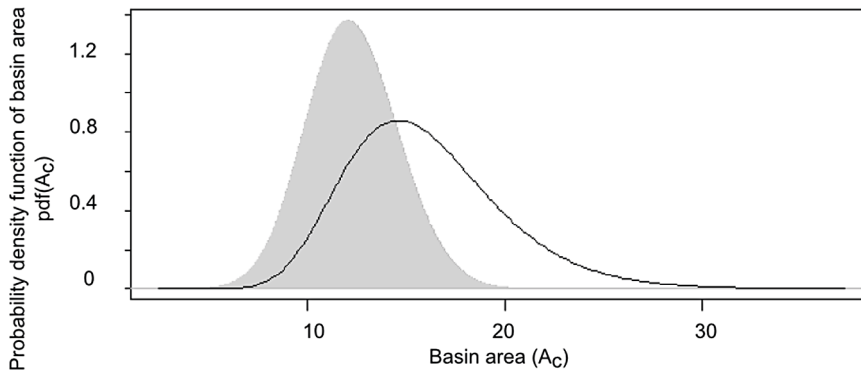
Fig. 3.3. Example of prior (dark solid line) and posterior (gray polygon) PDF for basin area (Ac)

The analysis of pdfs aims at assessing the gain of the information contained in data. This is usually assessed by two factors: a pdf width reduction and a shift of posterior towards prior pdf. Generally, if the posterior becomes narrower than the prior, the learning process was successful. Opposite, the posterior wider than the prior indicates that the prior was too confident (too narrow) for the information contained in the data given the model. The posterior similar to the prior states that there was no gain of information from the data content. This may lead to the identifiability problem (Sect. 2.1.1). A significant shift of the posterior marginal towards the prior states about the relevance of the learning process.

Parameters correlations

The posterior contains information on mutual correlations between model parameters θ^M. This means that, even if for the prior independence between all parameters is assumed, the posterior will contain dependencies between the parameters. The correlation between parameters can be assessed graphically by plotting posterior samples for all parameters against each other as a scatter plot.

3.4.3. Measure of predictive uncertainty

Residuals analysis

Model residuals, prediction errors, represent the difference between a predicted and an observed variable. The analysis of model residuals allows one to assess in how far the statistical assumptions underlying the introduced error model are fulfilled. This analysis should absolutely precede the analysis of derived predictive distributions, because only if the statistical assumptions are fulfilled, the derived uncertainty may be considered as meaningful.

It is sensible to evaluate the residuals at the maximum of the posterior, i.e. mode: $p(\tilde{Y})$ (e.g. Reichert 2011). $p(\tilde{Y})$ is the most likely model prediction which can be interpreted as the best model prediction and that would represent the best fit in a traditional deterministic approach. Usually, the residual analysis is performed during the calibration while predictive distributions are checked in the model validation.

Confidence and prediction intervals

To quantify the uncertainty of model predictions i.e. $p(Y)$, it is useful to compute the highest probability density regions. These can be computed as uncertainty bands expressed between the upper and lower uncertainty limits, which are defined as the prediction intervals (PIs) (Shrestha, Solomatine 2006). PIs usually refer to model predictions computed for future events, in validation. In the same fashion, PIs may be computed for model simulations in calibration period. In some application, it is useful to compute the confidence intervals or creditability intervals (CIs) which represent the uncertainty only due to the parameter uncertainty of a deterministic model. PIs are wider than CIs because they consider additionally input and output uncertainties.

PIa and CIa may be approximated by quantile ranges, typically as $(100-\psi)\%$ where ψ defines a range of the prediction intervals (Congdon 2003). Then PIs or CIs are defined as bands between lower and upper limits that correspond to $(\psi/2)\%$ for
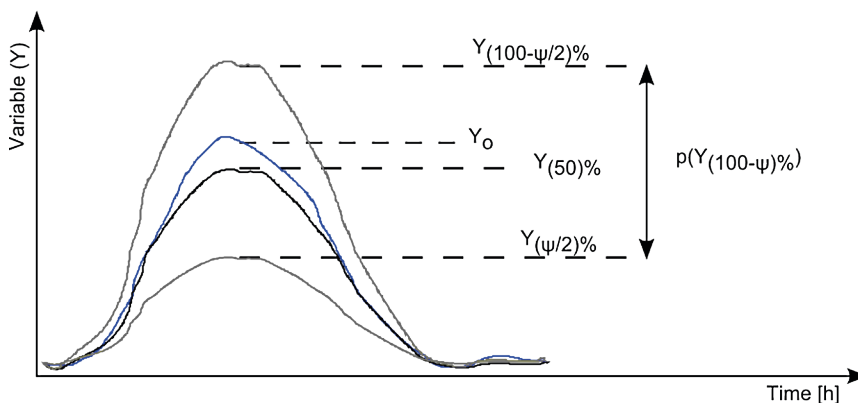
Fig. 3.4. Uncertainty intervals; Y_o is observed variable, Y is predicted variable, $p(Y(100-\psi)\%)$ are $(100-\psi)\%$ – predictions interval, $Y(\psi/2)\%$ and $Y(100-\psi/2)\%$ are lower and upper limits respectively, $Y(50)\%$ is the prediction median

lower and $(100-\psi/2)\%$ for upper limits (Fig. 3.4). Typically, PIs or CIs for 95% or 90% are computed. The 95% uncertainty bands provide with bands within which a predicted variable falls with 95% probability.

To summarize the predictive capability of the model, usually two uncertainty metrics are estimated (Montanari, Koutsoyiannis 2012; Del Giudice et al. 2013): i) data coverage and ii) sharpness. The first one indicates how many percent of the observation data are covered by the PIs or CIs. The closer (or greater) the percent of data coverage to the $(100-\psi)\%$ value is, the more reliable such PIs can be considered. For instance, for 95%-PIs the coverage of data should be ideally equal or higher than 95%. The sharpness measures the average width of the PIs. Ideally, the narrower bands are, the more useful they are. However, the bands have to still fulfill condition i) to be considered as reliable.

3.4.4. Model prediction performance

Because future events cannot be known, it is a common practice to assess the model performance by splitting the available data into two sets i.e. calibration and validation periods. Thus, a model is usually calibrated using part of all data and validated on the remaining data points, e.g. 10% or 20% (e.g. Haddad et al. 2013). Splitting data into two datasets requires enough observation data for both datasets. If the available data is limited or in case of event-based modelling, it is a standard practice to use a cross-validation instead. In general, this technique uses a (random) part of data set to calibrate model and the remainder to validate it (e.g. Haddad et al. 2013). Next, the procedure is redone for different parts of data so that the validation is executed for the whole data set. One of mostly applied cross-validation techniques is a leave-one-out cross validation method (e.g. Wang, Robertson 2011). This method uses all data set apart from a random single event (dataset) to calibrate the model and the remaining event to validate the model performance and is especially useful in event-based modelling. The leave-one-out cross validation method can be summarized in following steps:
1.   select a single event from all k available;
2.   use remaining events to calibrate the model;
3.   validate model on the event that was not used for the calibration;
4.   repeat steps 1-3 k-times so that each event is used to validate the model.

3.4.5. Predictive uncertainty decomposition

From Eq. 3.6, model predictions $p(Y)$ depend on pdf of model parameters $p(\theta^M)$. Treating all model parameters as a single vector θ^M is convenient if only the predictive distribution $p(Y)$ is of interest. However, one may be interested in knowing how the uncertainty of a particular model parameter denoted as θ^{Mj} contributes to the total $p(Y)$. In other words, how sensitive is the model and model predictions to changes in values of the θ^{Mj} (Saltelli et al. 2000). To this end, the computed predictive distribution may be conditioned on θ^{Mj} (Christensen et al. 2011) giving conditional distribution $p(Y|\theta^{Mj})$.

In practice, it can be achieved by keeping θ^{Mj} at the maximum of the posterior marginals (mode) and letting other parameters to vary within derived posteriors. Remaining parameters are then sampled from the posterior distribution conditional on the maximal posterior marginals of those parameters that are kept constant. So the Eq. 3.7 becomes:

$$p(Y|\tilde{\theta}^{Mj}) = \int p(Y^C|\theta^M)\, p(\theta^{M\text{-}[Mj]})\, d\theta^{M\text{-}[Mj]} \tag{3.12}$$

Where $\tilde{\theta}^{Mj}$ is the mode of the $p(\theta^{Mj})$, $p(Y|\theta^M)$ is the likelihood, $p(\theta^{M\text{-}[Mj]})$ is the parameter distribution when excluding the θ^{Mj} parameter.

By comparison with the full predictive distribution, one can indicate the relative importance of its components. Such a comparison of prediction uncertainty is preferable because it takes into account mutual parameters dependencies included in the posterior. In regard to hydrological flooding studies, it is more interesting to evaluate the importance of different uncertainty sources rather than the uncertainty arising from individual model parameters. Conceptually, Eq. 3.12 can be extended to assess contributions of parameter groups. To do so, a vector θ^M may be decomposed into its sub-vectors as: $\theta^M = (\theta^{MI}, \theta^{MII},..., \theta^{MZ})$. Where $\theta^{MI}, \theta^{MII},..., \theta^{MZ}$ represent now Z parameter subsets. The contribution of a particular sub-vector into the total $p(Y)$ may be then evaluated by conditioning the total predictive distribution on each of these sub-vectors.

3.5. Uncertainty consideration in hydrological modelling

3.5.1. Description of the model structure deficits

Gaussian error model
The classical assumption on E in Eq. 3.3 is that the model error is represented by a Gaussian error. This means that model residuals (see Sect. 3.4.2) are independent and identically (normally) distributed (i.i.d.) random variables with a mean of zero (Reichert 2011):

$$E(t_i) \sim N(\mu_E, \sigma_E^2) \quad \text{where} \quad \mu_E = 0 \tag{3.13}$$

Such an error term is usually introduced to lump together all uncertainty in hydrological modelling i.e. input, structural and output uncertainty. It is also usually assumed that E is constant over the time so that current model residuals do not depend on model errors observed in the past. This assumption is mathematically convenient and easy to implement and therefore has been widely used in applied hydrology (Yang et al. 2007). Unfortunately, this assumption was shown to be often violated in hydrological modelling since residuals of hydrological models are usually strongly auto-correlated (e.g. Romanowicz et al. 1994; Kuczera et al. 2006; Sikorska et al. 2012a, b; Wang et al. 2013). Intuitively, the residual autocorrelation is in an agreement with a RR process within a basin and can be explained by a basin memory effect due to current hydrological conditions. Therefore, model errors are expected to strongly depend on previously observed errors.

Lumped autoregressive error model

A promising alternative is a lumped continuous-time autoregressive error model (LEM), e.g. Brockwell, Davis (1996), Brockwell (2001), Yang et al. (2007), which is based on more realistic assumptions but has not been widely recognized so far (Del Guidice et al. 2013, Sikorska et al. 2012a). The LEM is especially useful to analyse time series e.g. streamflow data. It takes more justifiable assumptions than the Gaussian error model since it assumes that model residuals in the future intervals depend on the residuals that were already observed before. Residuals of the LEM are sometimes called as innovations. The relation between residuals of the LEM and Gaussian model is as follows:

$$I(t_i) = E(t_i) - E(t_{i-1}) \exp\left(- \frac{t_i - t_{i-1}}{\tau} \right) \tag{3.14}$$

$$\text{and} \quad I(t_i) \sim N(\mu_I, \sigma_I^2) \qquad \text{where } \mu_I = 0 \tag{3.15}$$

The LEM has two parameters: an asymptotic standard deviation (σ_I) and a characteristic correlation time of an error process (τ). The asymptotic standard deviation is described as:

$$\sigma_I(t_i) = \sigma_E \sqrt{1 - \exp\left(- 2 \frac{t_i - t_{i-1}}{\tau} \right)} \tag{3.16}$$

Where σ_E is a standard deviation of the error process E from Eq 3.13. If a time difference between observations ($t_i - t_{i-1}$) is large comparably to τ, than $\sigma_I = \sigma_E$. If $\tau = 0$, no correlation is assumed and LEM process becomes the independent Gaussian error as in Eq. 3.13. The LEM is continuous in time and therefore can also handle missing or irregularly spread data.

Such a LEM error model is sufficient to model the total predictive uncertainty (PU). However, if contributing sources are of interest, a separate treatment of uncertainty sources is recommended in order to quantify their contribution to the predictive uncertainty, see Sects. 3.5.2-3.5.3.

Variable transformation

Because in practice it is easier to deal with normally distributed errors, it is a common practice to apply a transformation function on variables (Yang et al. 2008; Wang et al. 2009; Sikorska et al. 2012a; Del Giudice et al. 2013; Honti et al. 2013). Thus, both variables can be transferred into the appropriate transformed space where transformed model errors can be assumed as i.i.d. and thus easier stabilized (Wang et al. 2013). This is particularly useful in hydrological modeling when the errors increase during high flow conditions (Del Giudice et al. 2014). A transformation function can be denoted as g; called also as a forward transformation. The transformed variable can be then written as:

$$z = g(y) \tag{3.17}$$

Where y and z are a variable and its transformed form respectively. Model errors E are modelled then as an additive random process to transformed modelled variables z. After adding the random error to the transformed results, a backward transformation (g^{-1}) is needed to the original scale for comparison with data (Yang et al. 2007, 2008):

$$Y = g^{-1}(z+E) = g^{-1}(g(y)+E) \tag{3.18}$$

Where y is the outcome of the deterministic hydrological model. The transformation functions g and g^{-1} are described by transformation parameters. Such parameters give degrees of freedom to improve the fulfillment of statistical distributional assumptions on model errors. The most widely used in hydrological modelling is the box-cox transformation in its one-or two-parameter forms (Yang et al. 2007; Wang et al. 2012), see Sect. 3.7. The two-parameter Box-Cox transformation is especially useful to deal with possible zero-values for a modelled or observed variable. If a variable takes non-zero values, the two-parameter Box-Cox transformation may be simplified to the one-parameter form. Alternatively, other transformations can be applied as e.g. Log-sinh transformation (Wang et al. 2012). The aim here is to use a transformation function that will allow one to normalize residuals and stabilize error variances. Usually preliminary analysis is required to evaluate different functions in order to choose the best transformation parameters.

3.5.2. Model parameter uncertainty

Formally, all model parameters can be captured into a single vector θ^M: $\theta^M = \{\theta^{M1}, \theta^{M2},..., \theta^{Mn}\}$, see Sect. 3.1.2. Thus, the uncertainty on the model parameters is accounted for by representing their values as a joint probability density function (pdf): $p(\theta^M)$. $p(\theta^M)$ represents the prior, pdf established before considering data at hand, whereas $p(\theta^M|Y^C)$ represents the posterior, prior updated with data. By the prior, $p(\theta^M)$, it is assumed that model parameters are independent (Christensen et al. 2011) and therefore usually pdfs of each θ^{Mi} are derived separately. Although, it is not always the case, numerically it is convenient to evaluate and therefore commonly applied in practice (e.g. Yang et al. 2007; Reichert, Schuwirth 2012; Honti et al. 2013). The computed posterior $p(\theta^M|Y^C)$, however, always contains mutual interactions between model parameters.

3.5.3. Input error model of precipitation

Usually in hydrological models, the model input X from Eq. 3.3 consists of input precipitation Px. Px contains errors mostly due to areal averaging and an inability to accurately capture a real precipitation which is spatially and temporally diverse (see Sect. 3.2.4). These errors cannot be known because the real precipitation is not measured. This uncertainty of input precipitation is modelled as proposed by Kavetski et al. (2006a, b). The input precipitation is tackled for each storm event with an

individual rainfall multiplier marked as ζ_j as illustrated in Fig. 3.5. For each rainfall event a unique multiplier is required because the accuracy of capturing rainfall fields may change from one to another rainfall event due to diverse characteristics of each rainfall, e.g. spatial and temporal variability, rainfall intensity. The product of ζ_j and the input precipitation Px marked as $P^\zeta x$ is then used as an input into the model instead of directly observed input rainfall Px.
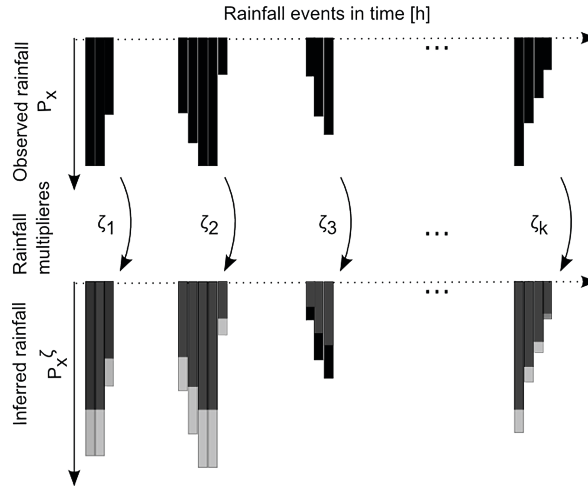
Fig. 3.5. Idea of the rainfall multipliers approach; Px − observed rainfall, ζ_j − rainfall multiplier for the j-th event, $P^\zeta x$ − inferred observed rainfall after including rainfall multiplier ζ_j, j − index of the rainfall event, $j = 1, 2,..., k$, k − number of analysed rainfall events

The rainfall multipliers cannot be known precisely beforehand. A priori, each ζ_j can be thus represented by the same probability distribution $p(\zeta)$. A vector ζ consists of all multipliers $\zeta = \{\zeta_{j1}, \zeta_{j2},..., \zeta_{jk}\}$ and can be further described as a random variable with an expected value of μ^ζ and a standard deviation of σ^ζ. These both parameters are integrated into a vector θ^ζ and prior uncertainty is described as $p(\theta^\zeta)$. Intuitively, an expected value of rainfall multiplier may be assumed as one. This is reasonable because if the error of a rainfall measure would be known in advance, observed input rainfall should be directly corrected before modelling.

This rainfall multipliers approach has been proved to lead to a better performance of a hydrological model and parameter estimation (Kavetski et al. 2002). However, it is only suitable for event-based modelling. The method becomes insufficient for continuous modelling because it requires a separation of observed data into rainfall events what is not always straightforward. Moreover, rainfall multipliers must be inferred together with θ^M during the calibration. Thus, the number of parameters to be now calibrated increases with the number of analysed events (k).

3.5.4. Consideration of calibration data uncertainty

The uncertainty in calibration data for rainfall-runoff models (output uncertainty) is considered by acknowledgement of the uncertainty in observed streamflow Q_o.

As stated in Sect. 3.2.4, Q_o is usually indirectly measured by converting a measured water level into „observed" streamflow Q_o usually by the use of a water level-runoff model that typically is a rating curve (RC). A standard RC is a power law equation and has three empirical parameters which can be closed in a vector $\theta^{RC} = \{\theta^{RC1}, \theta^{RC2}, \theta^{RC3}\}$. These parameters cannot be known precisely (Sect. 3.2). The uncertainty in θ^{RC} is considered by $p(\theta^{RC})$. This uncertainty propagates through the RC model and is further mapped onto estimated Q_o. Thus, $p(\theta^{RC})$ may describe the uncertainty of calibration data for RR models.

3.6. Example of uncertainty analysis application to SUB

In this Section the usefulness of the proposed uncertainty analysis is demonstrated on two practical examples, in which water level and streamflow in SUB are modeled, see Tab. 3.1. These two cases are complementary to each other and they together focus on estimating the PU in SUBs for flood risk studies. Specifically, within the example I (Sect. 3.6.1), the uncertainty in streamflow predictions and the contribution of the input (typically mean areal precipitation) and hydrological model parameter uncertainty to the total predictive uncertainty are evaluated. The input uncertainty is modelled explicitly by rainfall multipliers approach (Sect. 3.5.3). The uncertainty of water level predictions and the importance of the output uncertainty to the total PU are evaluated within the example II (Sect. 3.6.2). The output uncertainty describes the uncertainty in calibration data for RR models, which is typically derived with RC (Sect. 3.5.4).

Table 3.1

Focus of the predictive uncertainty analysis in two application examples

Application example	Uncertainty treatment				Focus of UA
	Structure[a)]	Parametric	Input	Output	
I	O	X	X	O	Input
II	O	X	O	X	Output

UA – uncertainty analysis. [a)] model structure deficits are implicitly modelled by the LEM; x indicates the uncertainty explicitly acknowledged, whereas o illustrates an implicit treatment of the uncertainty; lumped jointly into LEM

3.6.1. Example I – uncertainty of streamflow predictions

The first example illustrates a traditional approach in RR modelling when making predictions for future (unknown) events. To this end, the RR model is first calibrated against past recorded rainfall-streamflow data and next used to predict streamflows in response to some assumed rainfalls. Thus, in this example two uncertainty aspects are evaluated:
•   the total uncertainty of RR model predictions;
•   contribution of uncertainty sources (input vs. RR parameters).
    The assumption made here are as follows. Measurement errors of the calibration data for the RR model (typically streamflow), ε in Eq. 3.2, are significantly

smaller than other uncertainty sources. This can be assumed when a measure of the observed streamflow is said to be precisely, e.g. due to special efforts put into a maintenance of the equipment, calibration and verification of data. This is a common practice in hydrological modelling that uses streamflow data to calibrate RR models.

Given that, uncertainty sources are explicitly modeled by:

- Uncertainty of RR model parameters by $p(\theta^{RR})$ as described in Sect. 3.5.2.
- Input uncertainty is treated separately from the model bias B (Eq. 3.1) by tackling input precipitation Px with rainfall multipliers (ζ) of which uncertainty is described by θ^ζ (see Sect. 3.5.3).
- Measurement error of streamflow, ε, is lumped together with the model structure deficits to a single error term represented now by Em. Note that this error is marked as Em in order to distinguish it from E which lumps also input uncertainty (see Eq. 3.2). Em is described by LEM with parameters θ^{LEM}. To account for autocorrelated and not normally distributed errors of hydrological RR model, a likelihood function that combines the LEM with a Box-Cox transformation (see Sect. 3.5.1) is implemented as developed by Yang et al. (2007, 2008).

Treating input error explicitly allows one to assess weights of input vs. model parameter uncertainty at the next stage. To this end, an additivness of different uncertainty sources is assumed (Sect. 3.4.5).

Stochastic modelling of RR input

Input precipitation into RR model $P^\zeta x$ is modelled by correcting the observed input precipitation Px with rainfall multipliers ζ, see Fig. 3.6 (also Sect. 3.5.3). Thus Px is described as:

$$P_x^\zeta = P_x \cdot \zeta = R(P_x, \theta^\zeta) \qquad (3.19)$$

R is the input rainfall error model. ζ is modelled as a random variable of which probability is obtained by marginalising the joint probability distribution of input error model parameters i.e. θ^ζ and ζ:

$$p(\zeta) = \int p(\zeta | \theta^\zeta) \, p(\theta^\zeta) d(\theta^\zeta) \qquad (3.20)$$

Stochastic modelling of rainfall-runoff (RR)

The real streamflow Q is modelled as a sum of a deterministic RR model output q and an error term. This could be formalised accordingly to Eq. 3.3 as:

$$Q = q(P_x, \theta^{RR}) + E(\theta^{LEM}) \qquad (3.21)$$

Where $q(P_x, \theta^{RR})$ is a deterministic output from the RR model and $E(\theta^{LEM})$ accounts for model bias B (with input uncertainty) and measurement errors ε, compare with Eq. 3.2. Eq. 3.21 may be suitable if only the total PU is searched for. However, if the input uncertainty contribution is of interest, it must be explicitly acknowledged and separated from $E(\theta^{LEM})$:

$$Q = q(R(P_x, \theta^\zeta), \theta^{RR}) + E_m(\theta^{LEM}) \qquad (3.22)$$

Where $R(P_x, \theta^\zeta) = P_x{}^\zeta$, see also Fig. 13. $E_m(\theta^{LEM})$ lumps now only model structure deficits and measurement errors, while input uncertainty is represented by θ^ζ according to Eq. 3.19. A practical form and a magnitude of E_m cannot be known ahead and thus it must be assumed a priori, see Sect. 4.3 for practical recommendations. Availability of recorded data ($Q^C{}_o$) gives one the opportunity to update E_m during the Bayesian inference.
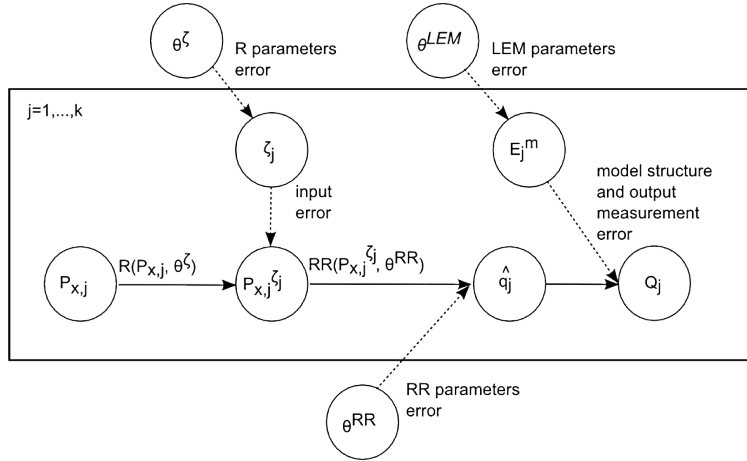
Fig. 3.6. Error model of the input uncertainty (R) and stochastic rainfall-runoff (RR) model; $j − j$-th rainfall event, $j = 1, 2,..., k$, $k −$ number of rainfall events, $P_{x,j} −$ observed rainfall for j-th event, $P^\zeta_{xj} −$ observed rainfall for the j-th event after including rainfall multiplier ζ_j, $\theta^{RR} −$ parameters of the RR model, $\theta^\zeta −$ parameters of the ζ, $\theta^{LEM} −$ parameters of the LEM model, $_j −$ modelled streamflow for j-th event, $E_m −$ error term for j-th event, $Q_j −$ real streamflow for j-th event, $t −$ time (over every event); dashed lines pinpoint error locations; bold font indicates a vector

Predictive uncertainty of Q

The probability distribution of the model output $p(Q)$ from Eq. 3.22 depends now also on θ^ζ. In the same fashion, the likelihood function of the model output depends on θ^ζ and is calculated as a joint distribution of the likelihood for the input error model $p(\zeta | \theta^\zeta)$ (Eq. 3.20) and the likelihood for the RR model $p(Q | \theta^{RR}, \theta^{LEM}, \theta^\zeta)$:

$$p(Q, \zeta | \theta^{RR}, \theta^{LEM}, \theta^\zeta) = p(Q | \theta^{RR}, \theta^{LEM}, \theta^\zeta) p(\zeta | \theta^\zeta) \tag{3.23}$$

For the notation simplicity and if only the total uncertainty analysis is of interest, all parameters are combined together into a single vector θ^I as $\theta^I = \{\theta^{RR}; \theta^{LEM}; \theta^\zeta\}$. Superscript I refers to the number of the application example. The uncertainty in θ^I is represented by $p(\theta^I)$. Thus, $p(Q)$ can be evaluated by marginalizing the joint distribution of Q and all parameters (Sect. 3.4.1):

$$p(Q) = \iint p(Q, \zeta | \theta^I) p(\theta^I) d\theta^I d\zeta \tag{3.24}$$

where $p(Q, \zeta | \theta^I)$ is the likelihood of Q (compare with Eq. 3.23).

Uncertainty contribution

An explicit treatment of diverse uncertainty sources in Eq. 3.24 allows one to assess how the total PU p(Q) is dominated by its contributing sources. Thus, Eq. 3.24 can be rewritten as:

$$p(Q) = \iiiint p(Q, \zeta | \theta^{RR}, \theta^{LEM}, \theta^{\zeta}) p(\theta^{RR}, \theta^{LEM}, \theta^{\zeta}) d\theta^{RR} \, d\theta^{LEM} \, d\theta^{\zeta} \, d\zeta \qquad (3.25)$$

To this end, an uncertainty analysis as described in Sect. 3.4.5 is undertaken in which either ζ or θ^{RR} are kept constant at their modes. For instance, assessing input uncertainty contribution would result in the following distribution:

$$p(Q|\tilde{\zeta}) = \iiiint p(Q, \tilde{\zeta} | \theta^{RR}, \theta^{LEM}, \theta^{\zeta}) p(\theta^{RR}, \theta^{LEM}, \tilde{\theta}^{\zeta}) d\theta^{RR} \, d\theta^{LEM} \, d\tilde{\theta}^{\zeta} \, d\zeta \qquad (3.26)$$

Where $\tilde{\zeta}$ and $\tilde{\theta}^{\zeta}$ refer to the best estimate (mode) of rainfall multiplier and its parameters for a particular rainfall event.

3.6.2. Example II – uncertainty in water level predictions

Introduction – problem in calibrating RR model

To improve model predictions, the RR model from Eq. 3.21 can be calibrated against recorded data that is typically Q^C for the RR. The calibration is usually complicated by the fact that the output of the RR model (Q) is not measured directly and must be derived from measured water levels L^C with help of another model (see Fig. 3.7 and Sect. 3.2.4). Consequently, a standard RR model calibration procedure consists of following steps (Sikorska et al. 2013):

$$\hat{Q} = LR(L_o, \theta^{LR}) + E^{LR} \qquad (3.27)$$

1. A water level L_{oi} is measured directly and a streamflow Q_{oi} indirectly e.g. by hydraulic measurements (see Sect. 3.2.4) for few temporal measured conditions.
2. Based on these temporal relations $L_{oi} - Q_{oi}$ a water level-runoff model (LR) that relates streamflow to the observed water level is constructed:
   where θ^{LR} is a parameter vector of the LR model and E^{LR} is the error term of the LR model and compensates here for all errors of the method (see Sect. 3.2.4). Usually it is assumed that E^{LR} is normally distributed around zero mean.
3. LR is calibrated to match the measured temporal streamflow records.
4. LR established on the step 3 is next used to obtain continuous streamflow data based on measured continuous water levels L_o using the best approximation of model parameters θ^{LR} and while neglecting E^{LR}:

$$Q = LR(L_o, \tilde{\theta}^{LR}) \qquad (3.28)$$

5. computed at the step 4 are next used as Q^C to calibrate the RR model. This can be formalized by comparing Q^C and Q from Eqs. 3.28 and 3.21:

$$\tilde{Q}(L_o, \tilde{\theta}^{LR}) = q(P_x, \theta^{RR}) + E(\theta^{LEM}) \qquad (3.29)$$

The described procedure might be useful to obtain the 'best fitting' parameters for the RR model (θ^{RR}). However, it has two conceptual drawbacks as for the uncertainty analysis of streamflow predictions. Namely, i) the error term of the LR model E^{LR} is 'lost' at the fourth step of the procedure and never 'seen' by the RR model; ii) the uncertainty in $\tilde{\theta}^{LR}$ is neglected. Even though, due to the error propagation, when RR model is calibrated against the average streamflow Q^C, computed with $\tilde{\theta}^{LR}$, E will contain also the uncertainty of the LR model so the 'lost' E^{LR}.
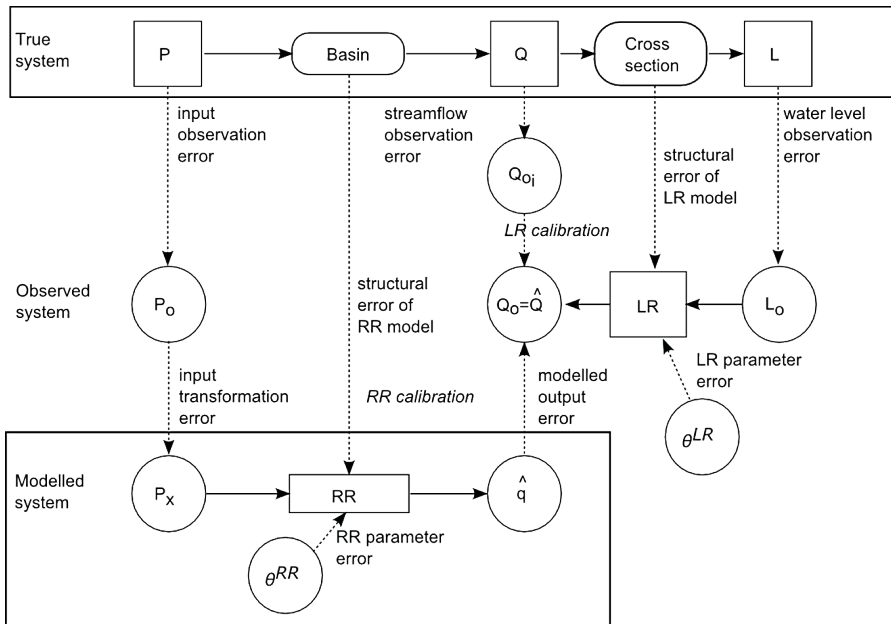
Fig. 3.7. Representation of the rainfall-runoff (RR) model calibration problem. Dashed lines pinpoint location of possible errors. Random quantities are shown in ellipsoids, deterministic in squares. Notation: P, Q, L represent real precipitation, streamflow and water level; P_o, Q_o and L_o represent observed variables; P_x is input precipitation into RR model; Q_{oi} represents measured streamflow records for LR calibration; $\hat{Q}$ is streamflow modelled with water level-runoff (LR) model; $\hat{q}$ is streamflow modelled with RR model; θ^{RR} – RR model parameters θ^{LR} – LR model parameters; bold font indicates vectors

Uncertainty analysis of water level predictions

In the example I (Sect. 3.6.1) it was assumed that the uncertainty in the calibration data for the RR model are much smaller than other uncertainty sources. However, as showed in Sect. 3.6.2, this uncertainty may be significant if the calibration data for RR models consist of streamflows derived from water levels usually with the use of a simple water level – runoff model which is typically a rating curve (RC). Therefore, the example II by directly modelling water levels with the runoff-water level (RL) model assess:

- the total uncertainty of RL model predictions;
- relevance of the output uncertainty by comparing RR vs. RC parameter uncertainty.

The assumptions made here are that water levels are modelled in two steps by RL model which consists of two submodels RR and RC, see Fig. 3.8. Measurement errors of water level for the RL model, ε in Eq. 3.2, are significantly smaller than other uncertainty sources (see also Sect. 3.2.4).
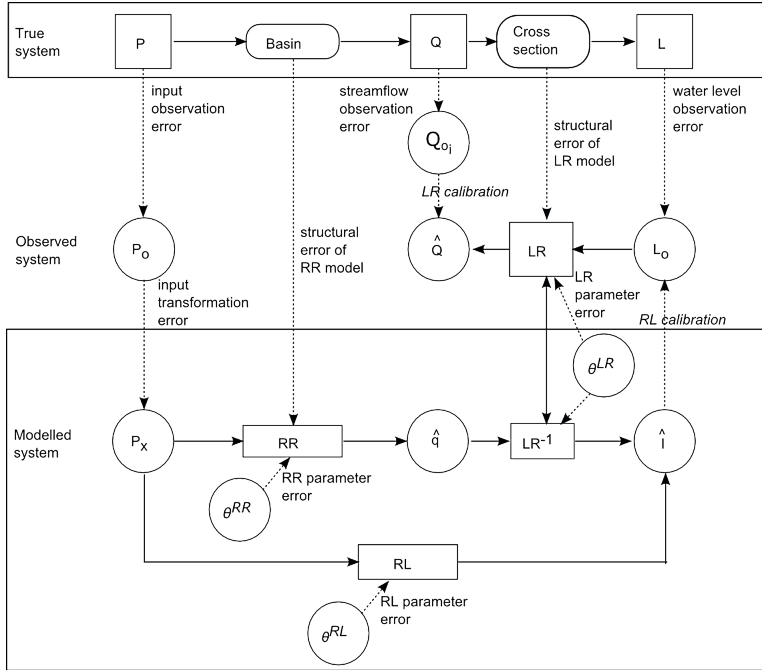
Fig. 3.8. Representation of the rainfall-water level (RL) model. Dashed lines pinpoint location of possible errors. ˆl is water level modelled with RL model. Other notation as in Fig. 3.9

Explicitly modelled uncertainty sources:
- Uncertainty in RL model parameters are explicitly described by RR and RC sub-model parameters as $p(\theta^{RR})$ and $p(\theta^{RC})$. RC refers to the rating curve.
- Input uncertainty is treated together with structural model uncertainty represented by model bias B (Eq. 3.2).
- Measurement error of water levels, ε, is lumped together with B to a single error term E, which is described by the LEM parameters θ^{LEM} (Sect. 3.5.1). A likelihood function which combines the LEM with a Box-Cox transformation is used to account for autocorrelated and not normally distributed errors of the RL model.

Treating RR and RC model parameters separately allows one to assess weights of RR vs. RC parameter uncertainty at the stage 2 and thus the relevance of output uncertainty for RR models.

Stochastic modelling of rainfall-water level (RL)
The real water level L is modelled as a sum of a deterministic RL model output l and an error term E accordingly to Eq. 3.2, see also Fig. 3.8:

$$L = l(P_x, \theta^{LR}) + E(\theta^{LR}) \tag{3.30}$$

where θ^{RL} is a vector with RL model parameters ($\theta^{RL} = \{\theta^{RR}; \theta^{RC}\}$). Although, conceptually similar to the RR transformation, this process usually is not straightforward to model because it involves an internal state of streamflow q that in such description is not modelled directly (Fig. 3.8). Therefore, in practice, RL consists of two submodels: rainfall-runoff and runoff-water level. The first part consists of the RR model as described in Eq. 3.21, while the second one usually is modelled by the inverse of a LR model from Eq. 3.27 and thus noted as LR^{-1}:

$$L = LR^{-1}(q(P_x, \theta^{RR}), \theta^{RC}) + E(\theta^{LEM}) \tag{3.31}$$

where the error term E lumps input uncertainty, measurement uncertainty of L and model structure deficits of both sub-models RR and RL^{-1}. Because E cannot be known in advance it must be assumed a priori. Availability of recorded data (L^C) makes it possible to estimate E by means of the Bayesian inference. Note, however, that q is now an internal state of the RL model and is not modelled directly. Thus, a calibration process only leads to a better fit of l to L without considering a match of q to Q (Fig. 3.10). Consequently, inferred parameters of RR submodel most likely will not be identical with the parameters of RR model when the RR model is calibrated alone (Sect. 3.6.1).

Predictive uncertainty of L

To assess the PU of L, all parameter vectors i.e. θ^{RR}, θ^{RC} and θ^{LEM} are combined together to the θ^{II}; $\theta^{II} = \{\theta^{RR}; \theta^{RC}; \theta^{LEM}\}$. Superscript II refers to the number of application example. Then, the probability distribution of the LR model output can be described by marginalizing the joint probability distribution of L and all parameters:

$$p(L) = \int p(L|\theta^{II}) \, p(\theta^{II}) \, d\theta^{II} \tag{3.32}$$

where $p(L|\theta^{II}) = \mathrm{p}(L|\theta^{RR}, \theta^{RC}, \theta^{LEM})$.

Uncertainty contributions

By an explicit acknowledgement of both parameters θ^{RR} and θ^{RC} in Eq. 3.32, their contribution to the total PU may be addressed by means of the sensitivity analysis when either θ^{RR} or θ^{RC} are kept constant at their maximal probability values (modes). For assessing the RC contribution, $p(L)$ results in a following distribution:

$$p(L|\widetilde{\theta}^{RC}) = \iiint p(L|\theta^{RR}, \widetilde{\theta}^{RC}, \theta^{LEM}) \, p(^{RR}, \widetilde{\theta}^{RC}, \theta^{LEM}) \, d\theta^{RR} \, d\widetilde{\theta}^{RC} \, d\theta^{LEM} \tag{3.33}$$

3.7. Implementation

3.7.1. Implemented likelihood function

The likelihood is strictly required in the Bayesian inference in order to explore a defined prior and to deliver the predictive distribution of the model. Likelihood (function) describes the pdf of observing the data Y given the model M and model

parameters θ^M as $p(\theta^M)$. Then for each candidate parameter set θ^{Mi} from the defined prior parameter space $p(\theta^M)$ a likelihood $p(Y|\theta^M)$ for given observations Y can be computed. A value of the likelihood function is proportional to the probability that the observations could have been generated by the parameter set θ^{Mi} i.e. $p(Y|\theta^{Mi})$ (Congdon 2003).

In combination with the transformation function g, the following likelihood function results (Yang et al. 2007, 2008; Sikorska et al. 2012a, 2013):

$$p(Y \mid \theta^M, X) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} \exp\left(-\frac{1}{2} \frac{\left[g(y_{o,t_o}) - g(\hat{y}_{t_o}(\theta^M)) \right]^2}{\sigma^2} \right) \cdot \left| \frac{dg}{dy} \right| y = y_{o,t_o} \left| \cdot \prod_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi}} \frac{1}{\sigma \sqrt{1 - \exp\left(-2 \frac{t_i - t_{i-1}}{\tau} \right)}} \right. \right.$$

$$\cdot \exp\left(-\frac{1}{2} \frac{\left[g(y_{o,t_i}) - g(\hat{y}_{t_i}(\theta^M)) - \left[g(y_{o,t_{i-1}}) - g(\hat{y}_{t_{i-1}}(\theta^M)) \right] \exp\left(-\frac{t_i - t_{i-1}}{\tau} \right) \right]^2}{\sigma^2 \left(1 - \exp\left(-2 \frac{t_i - t_{i-1}}{\tau} \right) \right)} \right) \cdot \left| \frac{dg}{dy} \right| y = y_{o,t_i} \left. \right] \right]$$

(3.34)

where $y_{o;t}$ is an observation and $y_t(\theta^M)$ is a simulated model response at time t. Note that this form of the likelihood is valid for different transformations and only transformation functions g and g^{-1} change.

For estimation of rainfall multipliers an explicit likelihood must be defined and is given in Eq. 3.35.

$$p(\zeta \mid \theta^\zeta) = \prod_{i=1}^{j} \frac{1}{\zeta_i} \frac{1}{\sqrt{2\pi}\sigma_\zeta} \exp\left(-\frac{1}{2\sigma_\zeta^2} \left(\mathrm{h}\ \zeta_i - \mu_\zeta \right)^2 \right)$$

(3.35)

where i is the number of rainfall multipliers (and rainfall events).

3.7.2. Box-Cox transformation

A general (two-parameters) Box-Cox transformation (Box-Cox 1964, 1982; Yang et al. 2007; Sikorska et al. 2012a) can be written as:

$$g(y) = \begin{cases} \dfrac{(y + \lambda_2)^{\lambda_1} - 1}{\lambda_1} & if \quad \lambda_1 \neq 0 \\ \ln(y + \lambda_2) & if \quad \lambda_1 = 0 \end{cases}$$

(3.36)

$$g^{-1}(z) = \begin{cases} \dfrac{(\lambda_1 \times z + 1)^{1/\lambda_1} - \lambda_2}{\lambda_1} & if \quad \lambda_1 \neq 0 \\ \exp(z) - \lambda_2 & if \quad \lambda_1 = 0 \end{cases}$$

(3.37)

$$\frac{dg}{dy} = (y + \lambda_2)^{\lambda_1 - 1} \tag{3.38}$$

where y is the system outcome (observed Y or modelled y), z is a forward transfer-red system outcome. λ_1 and λ_2 are Box-Cox transformation parameters. Note that g includes the identity ($\lambda_1 = \lambda_2 = 1$) and a log-transformation ($\lambda_1 = \lambda_2 = 0$) as special cases. The two-parameters Box-Cox transformation is especially useful to deal with possible zero-values for a modelled or observed variable. Thus, λ_2 is set to non-zero value. Thus, not y but $y + \lambda_2$ must be grater than zero. In addition, z must be larger than zero for all values of z. If a variable takes non-zero values, the two-parameters Box-Cox transformation may be simplified to the one-parameter form by setting λ_2 to zero. So the one-parameters Box-Cox transformation can be written as:

$$g(y) = \begin{cases} \dfrac{(y)^{\lambda_1} - 1}{\lambda_1} & if \quad \lambda_1 \neq 0 \\ \ln(y) & if \quad \lambda_1 = 0 \end{cases} \tag{3.39}$$

$$g^{-1}(z) = \begin{cases} \dfrac{(\lambda_1 \times z + 1)^{1/\lambda_1}}{\lambda_1} & if \quad \lambda_1 \neq 0 \\ \exp(z) & if \quad \lambda_1 = 0 \end{cases} \tag{3.40}$$

$$\frac{dg}{dy} = (y)^{\lambda_1 - 1} \tag{3.41}$$

3.7.3. Uncertainty analysis and uncertainty bands

Bayesian uncertainty analysis can be easily implemented in R programming language (R Development Core Team 2011), which is an open-source and thus is competitive to the other paid programming languages. Computation of the posterior uncertainty bands requires numerical implementation which allows for evaluating the likelihood function. In practice, this is extremely difficult but there are currently available algorithms that allow approximating the posterior by sampling from it. To this end, Monte Carlo Markov Chain (MCMC) algorithms can be easily adapted. Currently, there are a few practical algorithms available which allow one for a sufficient sampling. One of those is the generic adaptive MCMC algorithm proposed by Haario et al. (2001) and implemented by Vihola (2012) based on the Metropolis sampling (see below). These samplers adjust the covariance matrix of the jump distribution of searched parameters to achieve a defined rejection rate and thus guarantees efficient sampling and fast posterior convergence.

Markov Chain Monte Carlo (MCMC) sampling

The most popular are Markov Chain Monte Carlo methods (MCMC) that sample repeatedly from the joint posterior of all parameters. The sampling is evaluated by creating a random walk (Markov Chain) through the search space of the parameter distribution based on the a'priori assumption pdf(θ) (Vrugt et al. 2008b). This means that to every position within the whole parameter space there is assigned a unique pair of parameter values. To explore this space, MCMC generates a trial of move (v) based on the present location (ω_{t-1}). This trail u is either accepted or rejected depending on chosen from a parameter space values (see Fig. B.2). For the associated parameter values with this trail v the model is run and for its response the likelihood function is evaluated and compared with the likelihood for observations. The difference between both is called a likelihood value Le and is a function that quantifies how well chosen particular parameter combination simulates the system. Higher values of the likelihood function typically indicate better correspondence between the model predictions and observations (Vrugt et al. 2008b). The cruel values within the MCMC are: a number of sampling, a scale factor of each parameter α, which develops the step between (ω_{t-1}) and (v), and initial values of parameters. The initial values affect the chain only in its initial part, which in case of inaccurately selected values, need to be separated from the main chain. This pre-phase is described as burn in effect (see fig. 3.11). Properly selected initial values allow achieving satisfactory results in a relevant short time.
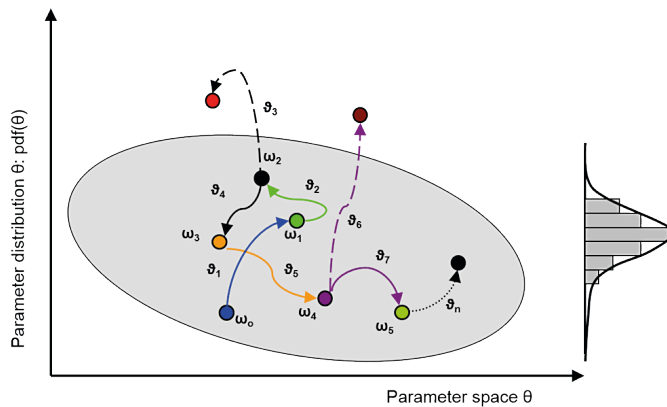
Fig. 3.9. A scheme of the Markov Chain Monte Carlo sampling;
solid line presents a move when accepted position, dashed line – when rejected, doted line – next evaluation, ω_n – current position, v_n – next proposed position, n – number of the move/position

MCMC-Metropolis sampling

Within the MCMC Metropolis algorithm the trial position is sampled from the proposal distribution $\pi()$. Next the trial move is either accepted or rejected depending on the metropolis acceptance probability (Fig. 3.11), where: $\pi()$ − density of the target distribution. If the trial is accepted the chain moves to its position (v), otherwise remains at the current location (ω_{t-1}), see also Fig. 3.9.

$$\alpha(\omega_{t-1},\nu) = \begin{cases} \min\left(\dfrac{\pi(\omega)}{\pi(\theta_{t-1})},1\right) & if & \pi(\omega_{t-1}) > 0 \\ 1 & if & \pi(\omega_{t-1}) = 0 \end{cases} \qquad (3.41)$$

Adaptive MCMC sampling

The main problem of the Markov Chain Monte Carlo techniques to sample from the posterior distribution is often a very slow convergence, especially when a prior is uninformative. The important advantage of the adaptive algorithm MCMC is therefore the possibility to permanently adopt the proposal distribution during the simulation run (Reichert 2011). That saves many unnecessary runs and allows for faster achievement of satisfactory results in a relevant short time, even if the initial values or a scale factor were not chosen properly.

Parallel MCMC chains

A possible extension is to run several Markov Chains in parallel and couple them adaptively. This leads to the posterior which summarizes results over all chains and usually allows for a better penetration of the parameters space. The limiting factor here is the computation time, which for many environmental models may be impossible to overcome.

Alternatively, other algorithms are available (Gilks et al. 1995; Haario et al. 2001; Brooks et al. 2011; Chievers 2012).

Monte Carlo Simulation

Because usually it is difficult to describe the predictive distribution of model's outcomes by statistics, it must be approximated. The most popular method to do so is a Monte Carlo simulation (MC). MC methods (or Monte Carlo experiments) are a class of computational algorithms that rely on repeated random sampling to compute their results (Fig. 3.10). MCs are commonly used to approximate predictive uncertainty intervals by performing multiple and numerous runs by randomly sam-
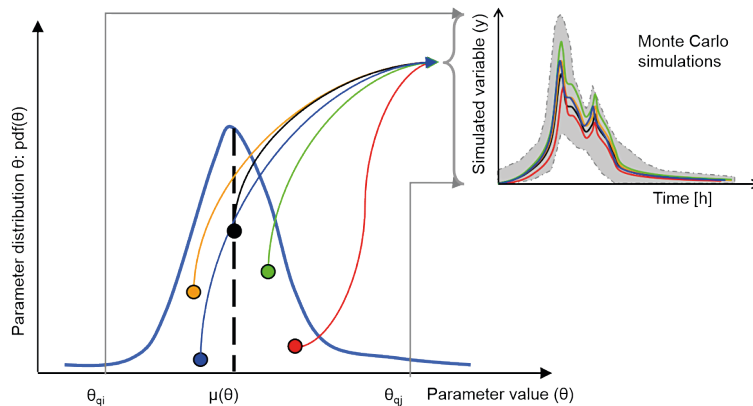
Fig. 3.10. Monte Carlo simulations; where *qi*, *qj* are *i*-(*j*-)quantiles

pling from the system space. The system space is defined as a space with all model possible solutions that may be obtained by running the model including parameters, model, input, output uncertainty.

3.7.4. Preliminaries

Necessary preliminaries include an elicitation of the parameter prior for the Bayesian inference and a choice of a transformation function for the modeled varia-ble. Ideally, a function that allows for the best fulfilment of statistical assumptions underlying the chosen likelihood function and provides with realistic uncertainty bands is searched for. Usually some preliminary analysis are required in order to chose the best solution. This could be easily undertaken within the R programming language.

In addition, a sufficient number and length of Markov's chains which will en-sure a good coverage of the parameter space should be chosen during preliminary trails. It is suggested to seek a compromise between a number of sample runs within the chain and computation time. Similarly, properly chosen initial values shorten the time required to fully explore the posterior. Those can be found with optimization methods. To minimize the effect of initial values, it is a common practice to cut away a burn in period at the beginning of the chain (Fig. 3.13).
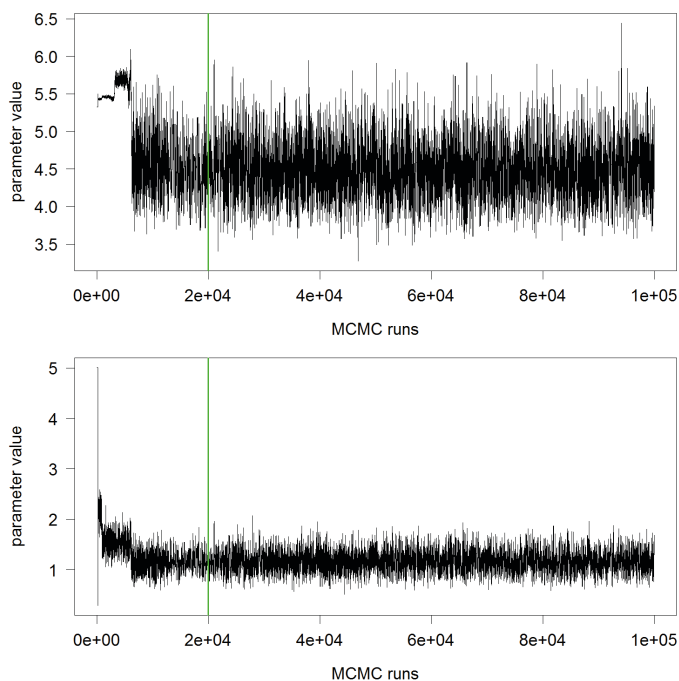
Fig. 3.11. Example of the MCMC chains; Black line – MCMC; X-axis – number of MCMC runs; y-axis – parameters values; green line – burn in; BOTTOM – uninformative and TOP – informative initial values

4. MATERIAL – RESEARCH BASIN AND HYDROLOGICAL MODEL STRUCTURE

4.1. Research basin

4.1.1. Overview

The proposed uncertainty analysis was tested on a small research basin, the upper part of the Sluzew Creek, located in south-west part of Warsaw, Poland (see Fig. 4.1). The Sluzew Creek basin, upstream of the investigated gauge „Wyscigi Pond", has an area of 28.7 km^2 and is rather flat; the elevation varies from 95 m to 110 m above mean see level. The average annual precipitation in this part of Warsaw is about 520 mm and the average daily temperature varies from -3°C in January to +18°C in July (WAU 2002).
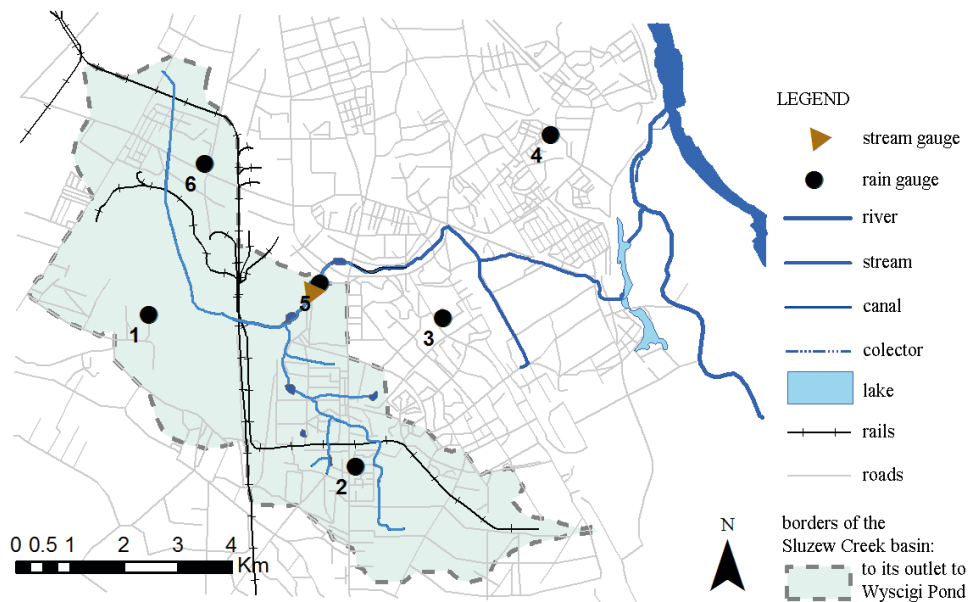
Fig. 4.1. Overview of the Sluzew Creek basin location, Warsaw

The Sluzew Creek basin was chosen for the study due to frequent flooding and flood-related sediment problems. In the last four decades Sluzew Creek has undergone rapid urbanization. Today urban areas cover 58.7% of the basin, whereas the ratio of impervious area of the whole basin is 32% (Sikorska et al. 2013). As a consequence, it is strongly affected by urban flooding (every second year) and associated sediment transport which mostly occur during the spring summer seasons (WAU 2002; Banasik et al. 2008; Sikorska, Banasik 2010; Sikorska et al. 2012). Unfortunately, no routine monitoring program exists and available data are limited to infrequent observations in the last few year. This is a typical case for a SUB (Sects. 2.1.3 and 2.2.4), where due to rapid changes within the basin, an adequate monitoring program has

not yet been established. This also strongly affects nature of hydrological modeling in SUBs (Sect. 2.3).

4.1.2. Measurement sites and available data

For the purpose of this study, a dedicated monitoring program was performed. It consisted of continuous measurements of precipitation at six locations across the basin, stream water level and temporal hydrometric measurements at the basin outlet (see Fig. 4.2).
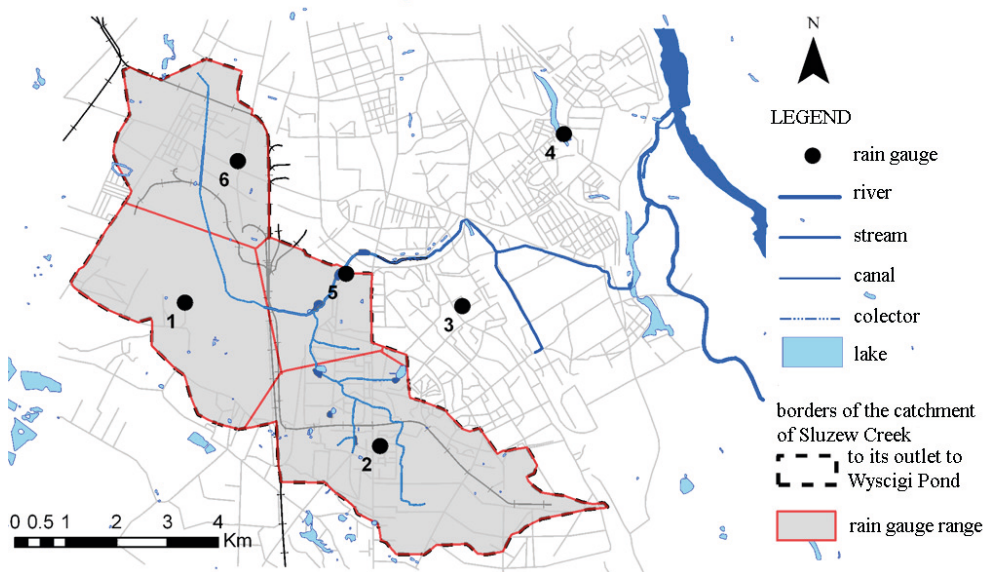
Fig. 4.2. Gauges and their contributing areas for six locations

Rain gauges: Precipitation has been measured at three different locations (sites 1-3 in Fig. 4.2) over the first three years (sites 1-3; July 2009 – October 2012). During the last period of the monitoring campaign (May – October 2012) three additional rain gauges were set up (sites 5-6 in Fig. 4.2). That gives in total three and a half hydrological years of measured precipitation with a temporal resolution of 10 minutes.

Stream gauge: A stream gauge has been installed at the outlet of the basin (see Fig. 4.1), here labelled as the Wyscigi Pond cross-section (WP). Monitoring program for the WP gauge included continuous measurement of water levels during three and a half years (July 2009 – October 2012), temporary measurements of a cross-sectional mean velocity during field experiments by means of the area-velocity method (WMO 2008) gathered regularly in intervals in 2010-2012.

Meteorological and hydrological data: As a result of the monitoring program, the following data were available for the purpose of the study: 35 rainfall-runoff (RR) events and 15 measurement points of area-velocity relations (see below).

RR events were selected based on the amount of total mean areal precipitation observed per event (>3 mm) and maximal streamflow observed during an event. A flow corresponding to more than three times the base flow over the analysed period was chosen as a threshold (>0.6 $m^3 s^{-1}$). Storm events with discontinuous rainfall and during winter periods, due to potential snowmelt that can significantly contribute to runoff, were excluded from the further analysis. This is justifiable since only rainfall-runoff events are of interest.
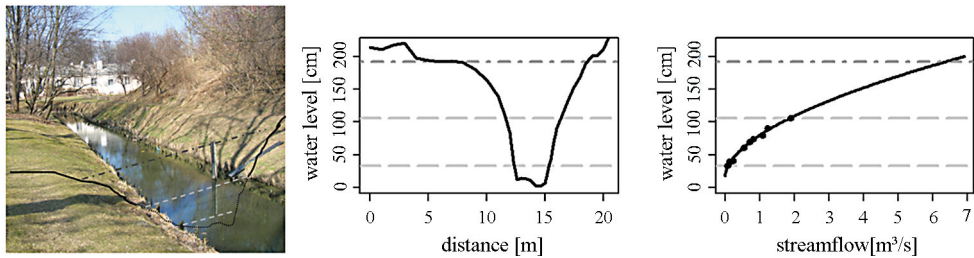
Fig. 4.3. Wyscigi Pond cross-section (left and middle), Sluzew Creek, and an empirical rating curve (right)

Cross-section hydrometric data: Fifteen temporal measures of water velocity vs. water level were gathered over the period 2010-2012. Based on those, an empirical rating curve (RC) was constructed using a power-law equation (see Fig. 4.3 right). All records were collected during spring-summer seasons. Therefore, the variation of the RC, although usually could be significant due to seasonal and alluvial changes within the channel, here may be assumed as irrelevant. RC was validated based on the information on the cross-section geometry collected via field measurements (Fig. 4.3 middle).

4.2. Hydrological model description

4.2.1. Conceptual rainfall-runoff model (RR)

A deterministic rainfall-runoff (RR) model transforms input precipitation Px into output streamflow q (see Eq. 3.22). For a typical event-based RR model this process consists of three main stages:
1.   estimation of the mean areal precipitation over the basin (Px);
2.   evaluation of the effective rainfall (EP); the rainfall available for runoff after excluding loss for infiltration and surface retention;
3.   routing of the EP to the basin outlet in order to determine the corresponding outlet streamflow (q).

The first part is usually external to the RR. Thus, most of RR models take already estimated Px as an input variable. These three stages are here specifically modeled as described in details below.

Mean areal precipitation

Px is estimated accordingly to the Thiessen polygons method (Thiessen, Alter 1911) in which the entire area of a basin (*Ac*) is divided into *n* rainfall fields or polygons (Ai) accordingly to station locations. The division between polygons is made by a line midway between the station under consideration and surrounding stations. A rainfall field is assumed to have the same precipitation as observed at the contributing station (*Pi*). The mean areal precipitation over the basin (*Px*) is determined from the weighted average of all defined rainfall fields where weights are their corresponding areas:

$$Px = \frac{\sum_{i=1}^{i=n} Pi \times Ai}{\sum_{i=1}^{i=n} Ai} \quad \text{and} \quad \sum_{i=1}^{i=n} Ai = Ac \tag{4.1}$$

This method, due to its simplicity and a practical value, has found widely applications in hydrological modelling (e.g. Montanari, Koutsoyiannis 2012). The limitations arise from no smoothing in estimated rainfall fields.

Effective rainfall

The EP is estimated from *Px* based on the Soil Conservation Sservice Curve Number (SCS-CN) method, called also NRCS-CN (Natural Resouces Conservation Service Curve Number, Hawkins et al. 2009). The SCS-CN method was developed by United States Department of Agriculture (USDASCS 1986, 1989). This method is frequently applied to evaluate the EP for RR models in small and poorly gauged basins (Banasik et al. 1988; Walker et al. 2000; Rosso, Rulli 2002; Mishra, Singh 2003; Hawkins et al. 2009; Soulis et al. 2009; Sikorska et al. 2012). The popularity of the SCS-CN method is caused, on the one hand, by including most of basins characteristics which produce runoff, such as soil type, land use and treatment, surface or antecedent moisture conditions. On the other hand, it has conceptual parameters that can be derived from physical properties of the basin. Therefore, it is feasible for modeling in small basins (Banasik et al. 2008; Hawkins et al. 2009; Soulis et al. 2009; Mishra, Singh 2010) and also urbanised ones (Sikorska, Banasik 2010; Sikorska et al. 2012a).

A cumulative effective precipitation EP(*t*) is computed here as:

$$EP = \begin{cases} 0 \ if \ \sum_{\varphi=0}^{t} Px(\varphi) - I \leq 0 \\[4mm] \dfrac{(\sum_{\varphi=0}^{t} Px(\varphi) - I)^2}{\sum_{\varphi=0}^{t} Px(\varphi) - I + S_{max}} \quad else \end{cases} \tag{4.2}$$

where $Px(\varphi)$ is the total mean areal rainfall at time $\varphi(t)$ estimated accordingly to Eq. 4.1, S_{max} is the maximal potential retention of the basin (mm), and *I* is the initial loss

(mm). I is usually event-specific and therefore difficult to estimate in advance. Thus, it is not modelled directly but assumed at a constant ratio of S_{max}: $I = \eta \times S_{max}$, where η is the ratio of S_{max} and for urban basins typically equals 5 %, whereas for rural equals 20% (Hawkins et al. 2009).

Streamflow at the basin outlet

The computed EP is convoluted into direct streamflow q at the outlet of the basin accordingly to the instantaneous unit hydrograph model (IUH) as proposed by Nash (1957). The concept of IUH was primarily developed by Sherman (1932) who defined the unit hydrograph (UH) as the direct runoff hydrograph resulting from a unit volume of effective rainfall of constant intensity which is uniformly distributed over the drainage area. The fundamental assumptions are that there is a linear relation between the inflow (input) and outflow (output) and that effective rainfall is uniformly distributed over the entire river basin. These assumptions are justifiable in small basins. A general form of the Unit Hydrograph $h(t)$ is described as:

$$h(t, \Delta t) = \frac{Ac}{3.6 \times \Delta t} \int_{t-\Delta t}^{t} u(\varphi)d\varphi \qquad [\text{m}^3\text{m}^{-1}\text{mm}^{-1}] \qquad (4.3)$$

where Δt is an interval time, t is continuous and φ is discrete time, and $u(\varphi)$ are the ordinates of the IUH at t. 3.6 is the units conversion factor. Eq. 4.3 is valid for diverse forms of IUH. Specifically within the Nash's model (Nash 1957), the IUH is represented as a gamma probability distribution function and is described:

$$u(t) = \frac{1}{k\Gamma(N)} \left(\frac{t}{k}\right)^{N-1} e^{-t/k} \qquad [h^{-1}] \qquad (4.4)$$

The parameters N [-] and k [h] describe a basin as a cascade of N linear reservoirs with a retention parameter k of each reservoir (see Fig. 4.4.).
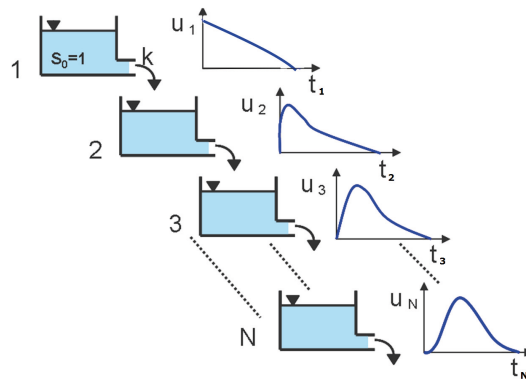

Fig. 4.4. Concept of a basin within the Nash's model;
IUH pools' areas are always constant and equal to 1.

EP(t) from Eq. 4.2 is next propagated through the cascade of N-reservoirs to produce q(t) at the basin outlet:

$$q(t) = \sum_{j=1}^{\min(i,n)} \Delta EP_j \times h(t, \Delta t) \qquad [m^3 s^{-1}] \tag{4.5}$$

The parameters of the RR model are described by a single vector θ^{RR}: $\theta^{RR} = \{Ac, S_{max}, k, N\}$. Because all θ^{RR} can be derived directly from the basin properties, a direct calibration of RR is not strictly required to provide streamflow estimates. The described RR model was used in two applications: in the example I to model streamflow (Sect. 3.6.1) and in the example II as a submodel of the rainfall-water level model (Sect. 3.6.2).

4.2.2. Rainfall-water level model (RL)

A rainfall-water level (RL) model introduced in Sect. 3.6.2 consists of: 1) the RR submodel as described in Sect. 4.2.1; and 2) a runoff-water level (LR^{-1}) submodel, presented below.

Runoff-water level submodel
The LR^{-1} submodel is the inverse of the water level-runoff (LR) model (Eq. 3.28). Here, a power law equation (e.g. Petersen-Øeverleir 2004; Di Baldassarre, Claps 2011; Domeneghetti et al. 2012; Le Coz 2012) has been used, which for the uniform cross section may be written as a classical rating curve (RC):

$$q(t) = RC1(l(t) − RC2)^{RC3} \qquad [m^3 s^{-1}] \tag{4.6}$$

$RC1$; … ; $RC3$ are empirical parameters of the RC represented by a vector $\theta^{RC} = \{RC1, RC2, RC3\}$. The LR model requires, however, the inverse form of the RC, so RC^{-1}, which for a predicted water level l can be written as:

$$l(t) = \left(\frac{q(t)}{RC1}\right)^{\frac{1}{RC3}} + RC2 \qquad [m^1 s^{-1}] \tag{4.7}$$

The parameters of the RL consist of the RC and RR submodel parameters (Eqs. 4.6 & 4.7) i.e. $\theta^{RL} = \{Ac, S_{max}, k, N, RC1, RC2, RC3\}$. The RL model was applied to model water levels in the example II (Sect. 3.6.2).

4.3. Prior knowledge elicitation

Bayesian inference requires an explicit formulation of the prior on all parameters as a probability density function. The aim is to find the distribution that best reflects the current knowledge. Unfortunately, this is not an easy task because no

explicit rules exist (O'Hagan 1998; Scholten et al. 2013). Therefore, for the purpose of this study, an approach to establish a prior has been developed. A prior on the deterministic models is elicited by means of the parametrization process (Sect. 2.2.2), whereas for the error models (Sects. 3.5.1 and 3.5.3) it takes advantage of expert knowledge and previous experience (studies). The approach is directly transferable to other research studies where the same models are applied. In a similar fashion, a prior on other hydrological model parameters with physical meaning could be obtained.

Since it is intricate to describe dependencies between prior parameters beforehand, an independence between all parameters is assumed a priori as done by Yang et al. (2007), Reichert and Schuwirth (2012) or Honti et al. (2013).

4.3.1. RR model parameters

In the following, the probability distribution function $p(\theta^{RR})$ is defined, $\theta^{RR} = \{Ac, S_{max}, k, N\}$ (Sect. 4.2.1). Ac represents the area of a basin in [km^2] and can be derived from topographic maps or GIS data. S_{max}, the maximum potential retention capacity of the basin [mm], is related to the Curve Number (CN) (USDA-SCS 1986, 1989) as:

$$S_{max} = 25.4\left(\frac{1000}{CN} - 10\right) \tag{4.8}$$

CN [-], represented as a function of basin land-use types, soil groups and hydrological conditions, is estimated as the average value over the entire basin according to the empirical values investigated by USDA-SCS. Usually CN can be derived from GIS data. An error of 10% of the estimated mean due to inaccurate maps may be assumed for both Ac and CN (Sikorska et al. 2012a). If no GIS data are available for SUB, these values must be elicited from topographic maps. However, while Ac usually remains constant for a basin over time, S_{max} may alternate. Thereto a sufficient wide prior distribution on S_{max} should be used.

k [h] and N [-] are Nash's model parameters and their average values may be derived directly via different empirical methods or indirectly through a relation to IUH characteristics (t_p & u_p). The relation between N, k and t_p, u_p are as follows:

$$t_p = k(N-1) \qquad [h] \tag{4.9}$$

$$u(t) = \frac{1}{k\Gamma(N)}\frac{(N-1)^{N-1}}{e^{N-1}} \qquad [h^{-1}] \tag{4.10}$$

where Nk = Lag, which describes the lag time [h]. Such empirical methods link parameters values to basin characteristics. However, their results may be biased. Therefore, it is proposed to use in parallel several empirical equations in order to infer parameter values. These values can be used to establish a prior, e.g. by the method of moments (Sikorska et al. 2012a). Here, five different empirical methods presented in Table 4.1 are applied simultaneously to fit the prior distribution for N and k. The

choice of applicable methods was dictated by its usefulness also for basins with poor information and also due to popularity of these methods in Polish conditions. The approach could be extended to other relations including exploratory analysis of the hydrographs (Haan et al. 1994; Bhunya et al. 2003; Jain et al. 2006; Singh 2007).

<div align="right">Table 4.1</div>

Methods to derive IUH characteristics and Nash model parameters

Method	tp or k	up or Lag
1. SCS (USDA-SCS 1986)	$t_p = \dfrac{L^{0.8}\left(\dfrac{1000}{CN}-9\right)^{0.7}}{2.92 J_z^{0.5}}$	$u_p = 0.75\dfrac{1}{t_p}$
2. Lutz (Lutz 1984)	$t_p = P_1\left(\dfrac{LL_c}{J_g^{1.5}}\right)^{0.26} e^{-0.016U} e^{0.004W}$	$u_p = P_2\dfrac{1}{t_p^{P_3}}$ i)
3. Rao (Rao et al. 1972)	$k = 0.56 A^{0.39} P_e^{-0.11} D_e^{0.22}(1+U)^{-0.62}$	$Lag = 1.28 A^{0.46} P_e^{-0.27} D_e^{0.37}(1+U)^{-1.66}$
4. GIUH (Rogríguez-Iturbe, Valdes 1979)	$t_p = 0.44\left(\dfrac{R_A}{R_B}\right)^{0.55} R_L^{-0.38} \dfrac{L_\Omega}{v}$	$u_p = 1.31 R_L^{0.43} \dfrac{v}{L_\Omega}$ ii)
5. GCIUH (Nowicka, Soczyńska 1989)	$t_p = 0.33\Pi^{0.67}$ $\Pi = \dfrac{L_\Omega^{2.5} n^{1.5} B_\Omega}{S_\Omega^{0.75} R_L^{0.6} A_\Omega i_r t_r}$	$u_p = 1.53\dfrac{1}{\Pi^{0.67}}$

i) $P_2 = 0.64$, $P_3 = 1.04$ [Lutz 1984]

ii) $v = 0.665 \cdot \alpha_\Omega^{0.66} \cdot (i_r \cdot A)^{0.4}$, $\alpha_\Omega = \dfrac{S_\Omega^{0.5}}{nB_\Omega^{0.67}}$ in (m^{-1}s$^{-1/3}$) and $R_B/R_A = 0.8$ [Rodriguez-Iturbe et al. 1982; Hall et al. 2001]

Notes: L – lenght of the stream to the central point, assumed to be equal to 0.5 l, U and W – ratio of urbanized and forest areas (%), P_1 – parameter dependent on the roughness of the stream, P_2 and P_3 – dependent on the interval of estimation, Lag – Lag time (h), A – total basin area (km^2), U – fraction of the impervious area in the basin (-), P_e and D_e – amount (mm) and duration (h) of effective rainfall, i_r and t_r – effective rainfall intensity (cm h^{-1}) and its duration (h), A_Ω, B_Ω, L_Ω – area (km^2), width (m) and length (km) of the highest order stream, R_A, R_B and R_L are the Horton area, bifurcation and length ratios of the basin [Tarboton 1996], v – average peak flow velocity (m s^{-1}), n – Manning roughness coefficient (m$^{-1/3}$s^{-1})

4.3.2. RL model parameters

The vector θ^{RL} consists of two sub-vectors θ^{RR}, as above, and $\theta^{RC} = \{RC1,\ RC2, RC3\}$ (Sect. 4.2.2). RC parameters should be ideally defined from some

field observations. For this purpose already existing data obtained from hydrometric measurements of cross-sectional average velocities and corresponding water levels can be easily adapted (Sikorska et al. 2013). Thus, it is proposed to calibrate the *RC* according to Eq. 4.7 with the standard Maximum Likelihood Estimate (*MLE*). MLE is a standard non-Bayesian technique to estimate single parameter values that give the best fit to the observed variable. In practice, *MLE* simply maximizes the likelihood function (Christensen et al. 2011). Next, $p(\theta^{RC})$ can be derived using large sample size properties of the *MLE* (e.g. Harrell 2010). A clear merit of using *MLE* is that such a prior contains information on mutual correlations within θ^{RC}. If no hydrometric measurements are available, one can consider constructing the *RC* according to the information on cross section geometry which, depending on the cross-section, can be obtained already from a single field inspection.

4.3.3. Lumped error model parameters (LEM)

The elicitation of the LEM model parameters θ^{LEM} (Sect. 3.5.1) is intricate due to the fact that they do not have a direct physical meaning. Mostly, because θ^{LEM} compensates for all errors not explicitly acknowledged in the study. Thus, θ^{LEM} differs in both examples. Namely, θ^{LEM} represents a combination of model structure errors and streamflow measurement errors in the example I (Sect. 3.6.1); and model structure and input precipitation error in the example II (Sect. 3.6.2). To represent such lack of knowledge on LEM parameters it is recommended to select wide positive distributions; e.g. gamma or log-normal distributions (see Table 5.2 for an example).

4.3.4. Input rainfall error model parameters

For urban basins, a log-normal distribution with a mean *1* has been suggested as a good prior on θ^ζ (e.g. McMillan et al. 2011; Sikorska et al. 2012a):

$$\zeta \sim LN(\mu_\zeta; \sigma_\zeta^2) \quad \text{where} \quad \mu_\zeta = 1 \tag{4.11}$$

The standard deviation σ_ζ [-] can be assumed based on the information of rainfall measurements (its accuracy and representativeness).

5. RESULT EXAMPLE: BAYESIAN UNCERTAINTY ANALYSIS IN SUB
5.1. Results of the preliminary analysis

Elicited prior for the Bayesian inference
The resulting prior elicited for the experimental basin of Sluzew Creek (Sect. 4.1) for two models: rainfall-runoff (RR) and rainfall-water level (RL) in two application examples is presented in Table 5.1. In addition, Table 5.2 presents results of the parametrization process for two parameters of the conceptual RR model, *N* and *k* (Sect. 4.2.1). The correlation of both parameters is verified in Fig. 5.1. As can be seen, no significant correlation between both parameters appeared.

Table 5.1

Prior distribution for the Sluzew Creek basin

Parameter		Prior
Abbreviation	Meaning	Distribution; Mean; Standard deviation
Deterministic models		
Rainfall-Runoff (RR) model		
A	basin area, [km^2]	N; 28.3; 2.8
S_{max}	maximal potential retention of the basin [mm]	LN; 55; 33
k	retention time of a linear reservoir [h][a]	LN; 2.0; 1.0
N	number of linear reservoirs [-][a]	LN; 3.2; 1.0
Rating Curve (RC) model		
$RC1$	coefficient, or streamflow scale [–][b]	N; -7.5; 1.1
$RC2$	location parameter, or cease to streamflow-water level, in units of the water level, e.g. [cm][b]	N; 16.8; 6.6
$RC3$	exponent, linked to the type and shape of the hydraulic control [–][b]	N; 0.6; 0.1
Error models		
Lumped error model (LEM)		
σ_I	asymptotic standard deviation of errors [m^{3}s^{-1}], [cm], [mgl^{-1}][c]	Γ; 2; 2
τ	characteristic correlation time of the autoregressive process [min]	Γ; 300; 200
Input error model (R)		
σ_ζ	standard deviation of n rainfall multipliers [mm][d]	Γ; 0.1; 0.05
ζ_j	rainfall multiplier for each j from n rainfall events [-][e]	LN; 1; E(σ_ζ)

[a] Distributions of N and k were derived from the empirical methods presented in the table 5.2; [b] prior on RC parameters is described as a multivariate normal distribution; [c] units of σ_I are of the modelled variable and therefore diverse for both experiments; [d] n – number of selected rainfall-runoff events; [e] ζ_j relates to the standard deviation of each rainfall multiplier, identical for all multipliers. Distributions: N – normal; LN – lognormal; Γ – gamma

Table 5.2

Nash's model parameters, k & N, derived empirically for the Sluzew Creek basin and their prior distribution

Method	k [h]	N [-]
SCS	2.20	4.70
Lutz	0.63	3.63
Rao	1.74	2.16
GIUH	1.82	2.81
GCIUH	3.54	2.73
Prior	LN; 2.0; 1.0	LN; 3.2; 1.0

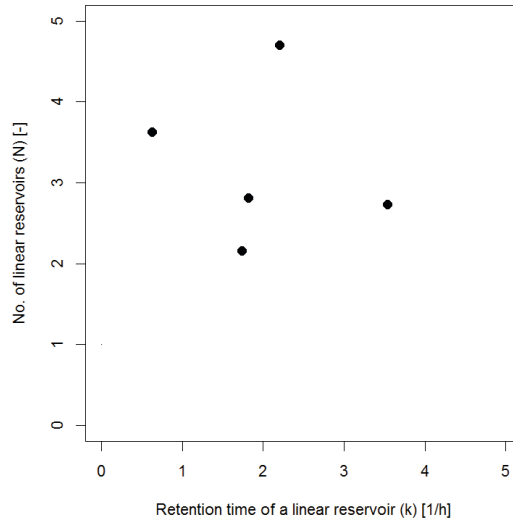Example I − streamflow predictions   65

Fig. 5.1. Correlation between N and k for five empirical methods

Parameters of transformation functions

In this study, the Box-Cox transformation was found to be sufficient in stabilizing model residuals. The transformation parameters which worked best were: $\lambda_1 = 0{:}35$ [-] for both examples and $\lambda_2 = 0$ [m³s⁻¹] for the example I and $\lambda_2 = 0$ [cm] for the example II.

Preliminary trials

Preliminary Monte Carlo Markov Chains (MCMC) were run in each application example in order to determine a sufficient number of chains and samples (see Sect. 3.7 for details). Namely, a chain with 1000 samples was run at first. Next, the number of samples was being gradually increased by doubling the previous number of samples until a good chain convergence was obtained. The resulting optimal number of samples were found to be 200 000 in the example I and 100 000 in the example II. In both examples, multiple chains were run in order to fully explore the posterior distribution; 12 and 3 respectively. All model parameters were inferred simultaneously.

5.2. Example I - streamflow predictions

In the example I, the uncertainty of streamflow predictions was assessed with a particular focus on the input uncertainty described by precipitation uncertainty.

5.2.1. Results of the statistical inference

Posterior parameter distribution

The inferred posterior parameter distribution $p(\theta^I|Q)$ is presented in Figure 5.2. The marginal posterior pdfs prove that the learning process from the data content

was beneficial for most parameters. Particularly, two RR parameters suggest that the response of the Sluzew Creek basin to the rainfall can be described by about 1.7 reservoirs (N) with a retention time (k) of 5.2 hours (Nash's parameters). For the other two RR parameters (A & S_{max}), the mode of the posterior, 3 km^2 & 3.2 mm respectively, was significantly shifted in comparison to the prior. A corresponds to the basin area, whereas S_{max} defines the maximal potential retention of the basin. Lower posterior values, although they may seem surprising, indicate that, first, in Sluzew Creek presumably only a fraction of the total basin area contributes to the surface runoff (A) during heavy rainfalls. Second, in regard to the natural retention of the basin (S_{max}), it does not seem to affect the surface runoff during heavy rainfalls. This can be justified because S_{max} is only significant for permeable sites; the retention of impermeable sites equals zero. A small value of S_{max} can thus be explained by the fact that during heavy rainfalls only the impermeable part of the basin contributes to the direct surface runoff, which is here modelled. This sounds reasonable for a SUB which is expected to have a rapid process of runoff formation (see also Sect. 2.1.2). The residual runoff from permeable sites is expected to be much postponed in time and is not explicitly modelled here.
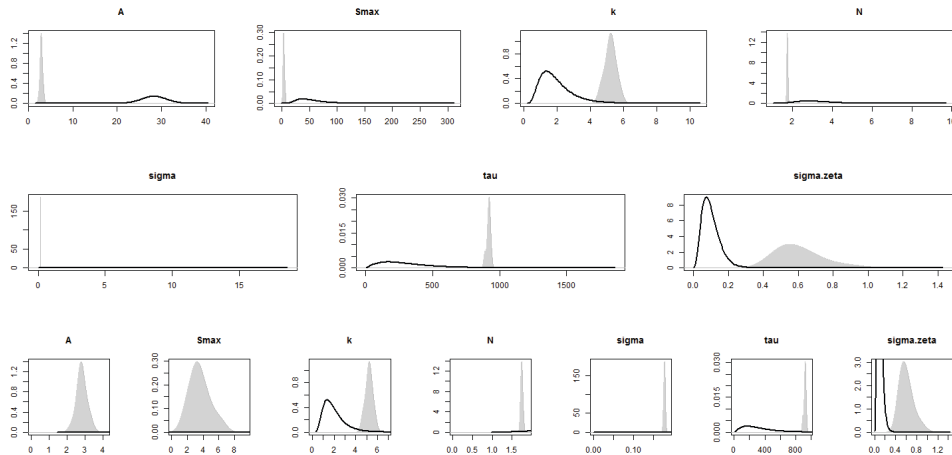
Fig. 5.2. Prior (solid line) and posterior (grey polygons) parameter distributions; A [km^2], S_{max} [mm], k [h], N [-] – RR parameters, σ_I [m^{3}s^{-1}] (sIM) and τ [min] – LEM parameters, σ_ς [-] – R model parameter. Y-axes – pdfs; x-axes – parameter values

As for the LEM parameters, much information was gained from the data because the posterior (modes: $\sigma_I = 0.2$ m^{3}s^{-1} & $\tau = 920$ min) is narrower and strongly shifted in regard to the assumed prior. The posterior parameter values should be interpreted as 'effective' values for the LEM in this basin, the applied RR model and this application example.

Regarding the input error model (R), interestingly, the posterior standard deviation of all rainfall multipliers (with a mode of $\sigma_\zeta = 0.55$) increased significantly

Example I − streamflow predictions 67

compared to the prior (0.1). This finding indicates that, first, the prior pdf underestimated the input uncertainty, and second, the deviations in input uncertainty among all rainfall events are high (see below). This could also be caused by the model sensitivity to changes in rainfall multipliers and is discussed below.

Parameters correlations

The correlations between all parameters are presented in Fig. 5.3. A significant correlation can be observed between two RR model parameters, *k* and *N*, which are the Nash's model parameters (see Sect. 4.2.1). They describe together the rainfall--runoff process within the basin and therefore their strong correlation should not surprise. Alternatively, one of the Nash's parameters could have been also kept constant while only the second was inferred. This could be considered if computation time was a limiting factor.

As can be seen, the inferred posterior contain mutual influences. Thus, model simulations should be always drawn from the full marginal posterior distributions. Sampling parameter values independently will lead usually to the overestimated uncertainty bands.
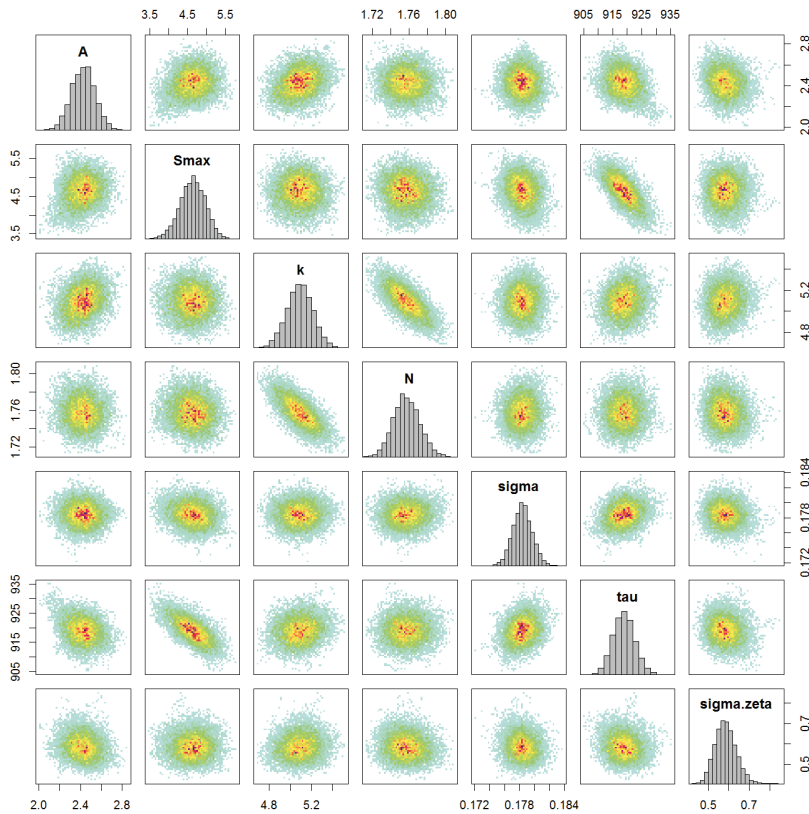

Fig. 5.3. Posterior parameters correlations in example I

Inferred rainfall multipliers

Figure 5.4 presents posterior pdfs for all 35 inferred rainfall multipliers.

A population of the inferred multipliers can be described by a mean which is close to the value of 1.0 [-] and a standard deviation of 0.55 (σ_ξ). The individual modes of the estimated rainfall multipliers varied from 0.82 to 3.2 with a mode of 1.3
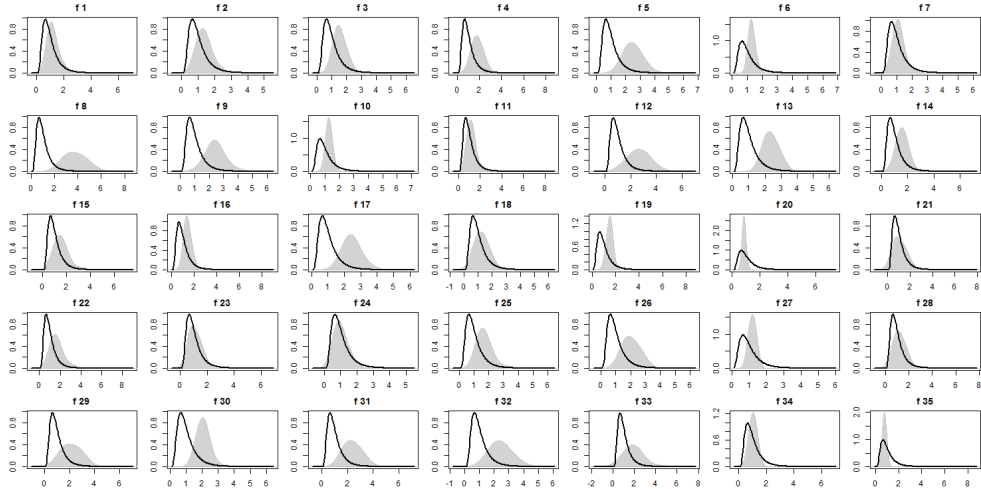
Fig. 5.4. Prior (solid line) and posterior (grey polygons) distributions of rainfall multipliers; numbers label rainfall-runoff events; y-axes – pdfs; x-axes – parameter values
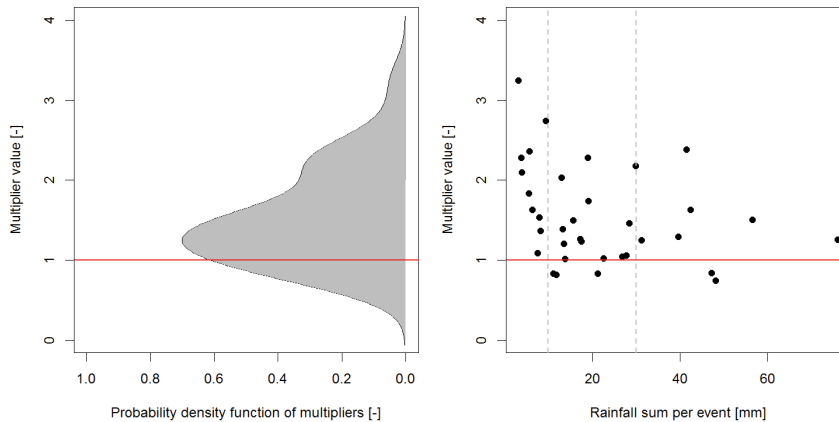
Fig. 5.5. Diagnostic plot of rainfall multipliers (z). left panel: posterior distribution $p(z)$ (solid line); x-axis – pdf; y-axis – values of rainfall multipliers [-]; right panel: black dots – relation between posterior modes of z (y-axis) and rainfall amounts observed per event (x-axis); dashed grey lines cut thresholds for small (<10 mm) and large rainfall events (>30 mm). Red dashed line – posterior mean of z

for all events. This nicely shows that the intuitive prior with a mean equal to 1.0 is on average a reasonable assumption. However, when analyzing each event separately,

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high deviations from this value were observed. For events with a higher observed rainfall, the accuracy of the rainfall measure was usually found to be higher than for those with a relatively small rainfall (Fig. 5.5). This can be well explained by two reasons. On the one hand, the precipitation measurement accuracy itself decreases significantly for small rainfalls due to the absolute equipment error. On the other hand, for small rainfalls the possibility that the measured rainfall is only locally observed and does not cover the whole basin increases. Therefore, when extrapolating measured values across the entire basin, the error may be considerable. This is further discussed in the Discussion, Sect. 6.1.

Fulfilment of statistical assumptions

The statistical inference of obtained results is crucial for the UA because only if statistical assumptions are fulfilled, the computed PU in calibration and validation mode can be considered as meaningful. The diagnostic plot of LEM model residuals is presented in Fig. 5.6.
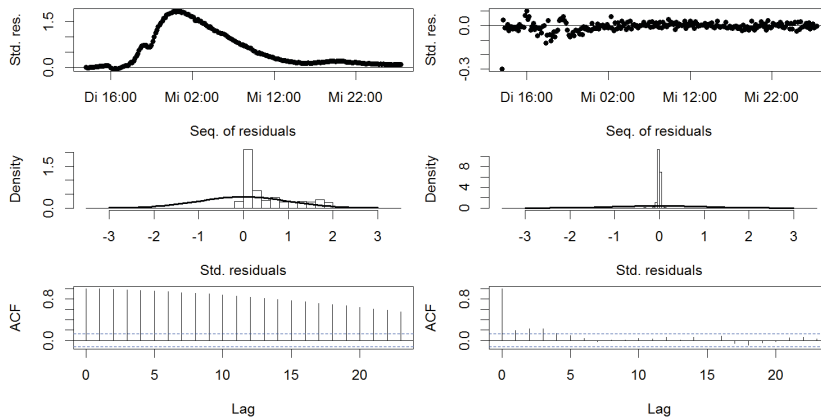

Fig. 5.6. Right panel, diagnostic plot of residuals in the LEM and, left panel, of residuals in the traditional Gaussian error model by the example of one event (No. 6) in example I.
The assumption of i.i.d. is clearly violated for residuals, middle left panel, and fulfilled for residuals of LEM, middle right panel. The top panels present sequences of residuals, left, and LEM residuals, right. A strong autocorrelation (ACF) can be observed for model residuals, bottom left.
ACF is significantly reduced for LEM residuals, bottom right

To assess the fulfillment of underlying statistical assumptions, the residuals of LEM are compared to the residuals of the traditional Gaussian error model. In both cases, residuals are computed as a difference between observed and simulated values corresponding to the best model prediction (mode). However, in LEM the difference between observed and simulated values are calculated in the transformed space; here after using the Box-Cox transformation.

As shown in Figure 5.6, the Gaussian assumption of i.i.d on residuals would have been here clearly violated because the residuals of the RR indicate a strong autocorrelation (bottom left panel) and a heteroscedasticity (top left). This means

residuals are not i.i.d., proved by middle left panel. Contrary, the assumption of i.i.d on residuals in the LEM is clearly better fulfilled (right middle panel) because innovations show strongly reduced autocorrelation (bottom right panel) and heteroscedasticity (top right panel), even if a slight autocorrelation still remains. This can be explained by structural deficits of the applied RR model which can not perfectly reproduce the observed variable; see Sect. 5.2.2 for further explanation.

5.2.2. Uncertainty of streamflow predictions

The predictive intervals (PIs) were computed for RR events in the validation mode by randomly sampling from the derived posterior with 1000 repetitions by means of the leave-one-out-cross-validation method (Sect. 3.4.3). This means that PIs for each event are the result from a model calibration without this event; 35 independent MCMC chains were generated for every calibration set of other 34 events and validated on the remaining one. Thus, 35 different full posterior distributions were computed. The resulting 95%-PIs are presented in Figure 5.7 as grey polygons. Solid red lines illustrate the best predictions for the mode of the posterior, whereas the validation data points are depicted with black dots.

The credibility of streamflow predictions was assessed accordingly to their: i) data coverage and ii) sharpness with respect to the observed streamflow data. It is worth noting that the observed streamflow records for each validated event are treated as reference ('future') data since they were excluded from the calibration mode.

Generally, 84% of data are properly covered by the PIs for all events, 14% of data lie above and 2% of data below the upper and lower limits, respectively. Lower
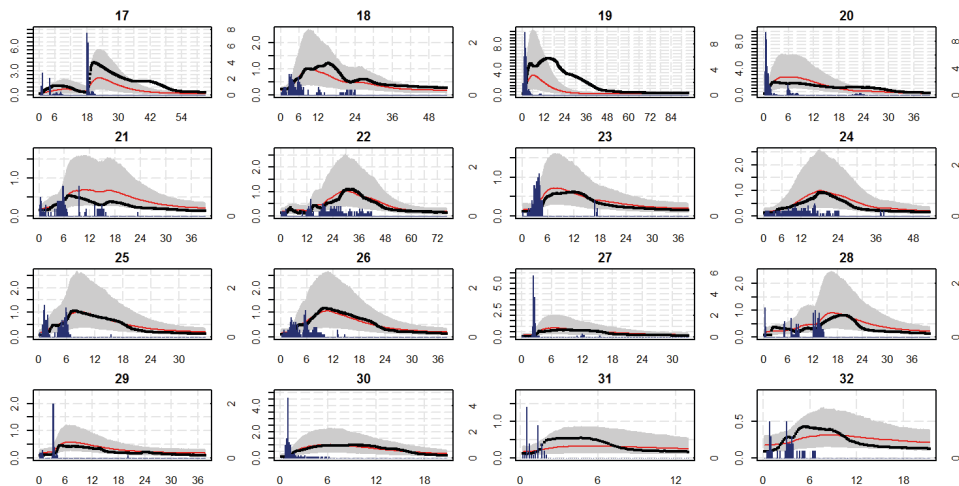
Fig. 5.7. 95% PI for predicted streamflows [m^3s^{-1}] in the Sluzew Creek in the validation mode. Bold numbers above events label rainfall events; x-axes represent time from the beginning of the rainfall event; y-axes – streamflow [m^3s^{-1}] and 10 minutes rainfall intensity [mm]. Dotted black line depict observations; red solid line – posterior mode; blue bars – observed rainfall; horizontal dashed grey lines are depicted in 0.5 intervals of y-axes values

Example I − streamflow predictions   71

coverage is observed for events No. 2, 8, 12, 13, 19. This is most probably due to limitations in the RR model structure that is restricted to model events which follow the conventional rainfall-runoff process within the basin.

The derived PIs are on average 113% and up to 350% higher than steamflow peaks during rainfall-runoff events; assessed by the upper limits.

5.2.3. Contribution of the input uncertainty

The relative contribution of the input uncertainty to the total PU was assessed by performing two independent MC simulations. First, PIs were derived by sampling randomly a parameter vector while RR parameters were kept constant at their maximum probability. Thus, the predictive uncertainty bands were derived whilst ignoring uncertainty in RR model parameters. Second, MC simulations were performed without considering the input uncertainty. Each rainfall multiplier was fitted to the value of 1.0 and no deviations were considered. As a next step, a parameter vector conditioned on fixed rainfall multipliers was sampled from the posterior. From this the resulting uncertainty bands without considering input uncertainty were derived. By comparison of both PIs to the total PIs, the importance of the input uncertainty vs. RR model parameter uncertainty was assessed and is presented in Fig. 5.8.

In general, the corresponding PIs while considering input uncertainty were found to be higher than those with neglecting input uncertainty. The latter bands are up to 65% and on average 30% narrower than the total PU bands; assessed by stre-
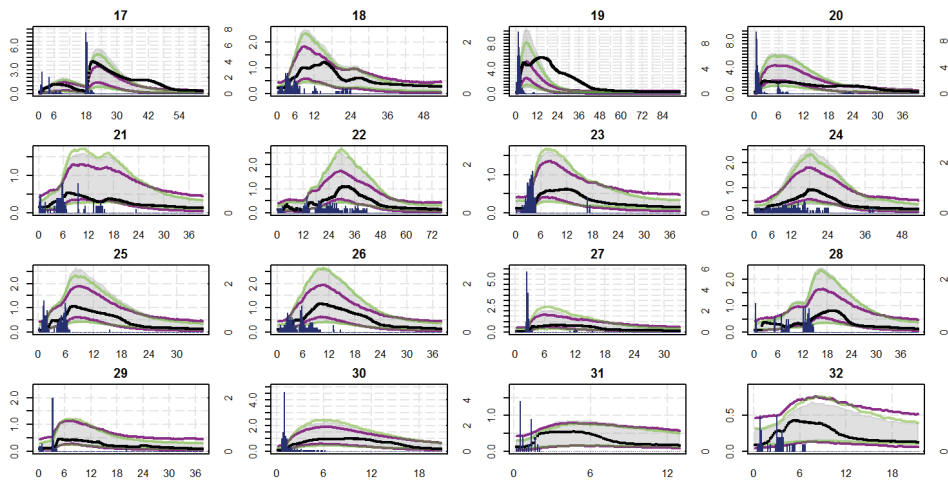
Fig. 5.8. influence of the input uncertainty in the Sluzew Creek streamflow [m³s⁻¹] predictions. Grey polygons describe 95% total PIs; green lines describe 95% PIs whilst ignoring the input uncertainty; violet lines present 95% PIs whilst accounting for the input uncertainty. Bold numbers above events label rainfall events; x-axes represent time from the beginning of the rainfall event; y-axes – streamflow [m³s⁻¹] and 10 minutes rainfall intensity [mm]. Dotted black line depict observations; red solid line – posterior mode; blue bars – observed rainfall; horizontal dashed grey lines are depicted in 0.5 intervals of y-axes values

amflow peaks. When parameter uncertainty is neglected the PIs become up to 25% and on average less than 0.1% smaller than the total PU bands. This finding would suggest that, first, the uncertainty due to input rainfall is more important (relatively 40% higher) than the uncertainty due to RR model parameters for streamflow predictions in Sluzew Creek. Ignoring input uncertainty would lead to narrower and thus presumably underestimated PI. Second, as can be noticed by comparison of both PIs, the LEM which lumps here model structure and measurement errors still contributes significantly to the total PU. This, however, is not straightforward to assess due to mutual correlations between parameters. For further explanation see Sect. 6.1.

The quantitative contributions of both uncertainty sources are always strongly case-related. Thus, the proposed UA should always be performed for new basins or research studies

5.2.4. Conclusions from the example I

The example I addressed the uncertainty of streamflow predictions in RR models and the relevance of the input variable uncertainty to the total predictive uncertainty (PU), vs. the RR model parameter uncertainty. Based on the results from the Sluzew Creek basin, the following conclusions can be drawn:

- The Sluzew Creek basin responds rapidly to heavy rainfalls and only a part of the basin contributes to the streamflow observed in the stream during heavy rainfalls. The PU of streamflows in Sluzew Creek is high and has on average value of 113% of the observed streamflow peak. The extreme uncertainty bands went up to a value of 350% higher than the streamflow peak. For the Sluzew Creek basin, the input uncertainty contribution was found to be relevant, up to 65% of the total PU, and higher than the RR model parameters uncertainty, which was up to 25%, respectively.
- The Bayesian approach was proved to be beneficial in assessing flood predictions in SUBs because it allows one for incorporating available knowledge in a feasible way and for an explicit treatment of diverse uncertainty sources. An explicit treatment of the input uncertainty adds value to the analysis. First, it avoids common assumptions on insignificant input variable error. Second, one can directly assess the input uncertainty contribution to the total PU. The adopted rainfall multipliers approach is very practical because i) it has manageable number of parameters to be inferred, ii) improves model fit to the data during the calibration mode, and iii) is feasible in making predictions.
- The example of Sluzew Creek shows that the Gaussian assumption of identically and independently distributed (i.i.d.) residuals does not hold for hydrological models because residuals are heavily autocorrelated. That can be explained by the memory effect of the basin and a simple hydrological model structure. Instead, the autoregressive continuous lumped error model, LEM, deals better with autocorrelated residuals by normalizing them in the transformed space. The fulfilment of the underlying statistical assumptions was satisfying for most of the analysed RR events. Thus, the LEM appears to be much more sufficient in explaining the RR process within the basin. Moreover, using a transformation function on variables allows for deriving more realistic uncertainty bands

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which, as intuitively expected, are higher for higher streamflows and vice versa, smaller for smaller streamflows. This is reasonable since more extreme events, for which less information is available, are expected to be more uncertain.

Several points need, however, to be further discussed. First, the not perfect fulfilment of the statistical assumptions underlying the LEM, which were observed for some of events, rises a question of a simplified conceptual models use. Second, although the rainfall multipliers approach appears to be very beneficial, it is limited to event-based modelling because a unique multiplier for each RR event must be specified. Third, a high contribution of the input uncertainty points out a high variability in rainfall fields. This variability cannot be captured by the traditional sparse rainfall gauging network and other methods should be considered. Fourth, also high uncertainty on predicted streamflows concerns question of a practical value of such high uncertainty in water management. See further Sect. 5.5.

5.3. Example II - water level predictions

In the example II the uncertainty analysis was applied to water levels modelled by means of rainfall-water level model (RL). In particular, the focus of the UA was put on assessing the importance of the output uncertainty represented by the rating curve parameters. These are relatively compared with the significance of RR parameters.

5.3.1. Results of the statistical inference

Posterior parameter distribution
As in the first application example, much information has been gained from the data content because the marginal posterior pdfs of both (deterministic) submodels
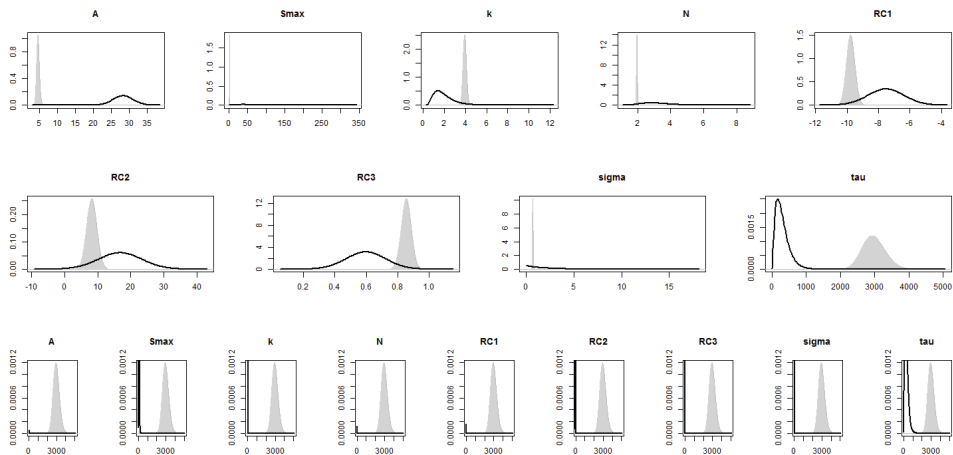

Fig. 5.9. Prior (solid line) and posterior (grey polygons) parameter pdfs; RR parameters: A [km^2], S_{max} [mm], k [h], N [-]; RC parameters: $RC1$ [-], $RC2$ [cm], $RC3$ [-]; and LEM parameters: σ_l [m^{3}s^{-1}] and τ [min]; y-axes − pdfs; x-axes − parameter values

i.e. RR and RC were smaller variances than the assumed prior. However, as before, the direct interpretation of inferred parameter values is not straightforward because these includes mutual dependencies. This also complicates a direct comparison with results obtained for RR parameter in the example I. Although inferred RR parameters were found to have slightly different values than in the previous example (Sect. 5.2.1), generally they further confirm the previous finding that during heavy rainfalls only a fraction of a basin area, which is heavily urbanized, contributes to flood flows. The detailed posterior for RR parameters are presented in Figure 5.9.

Not surprisingly, the posterior of the RC parameters is rather similar to the prior elicited. This was expected because rainfall-water level data do not contain information on the RC submodel parameters and therefore do not provide a significant learning process. Again, this finding emphasizes the importance of obtaining an informative prior distribution for the RC, as recommended in Sect. 4.3.

Finally, the learning process was beneficial for LEM parameters, similarly to the previous example. In both studies, however, the LEM compensates differently
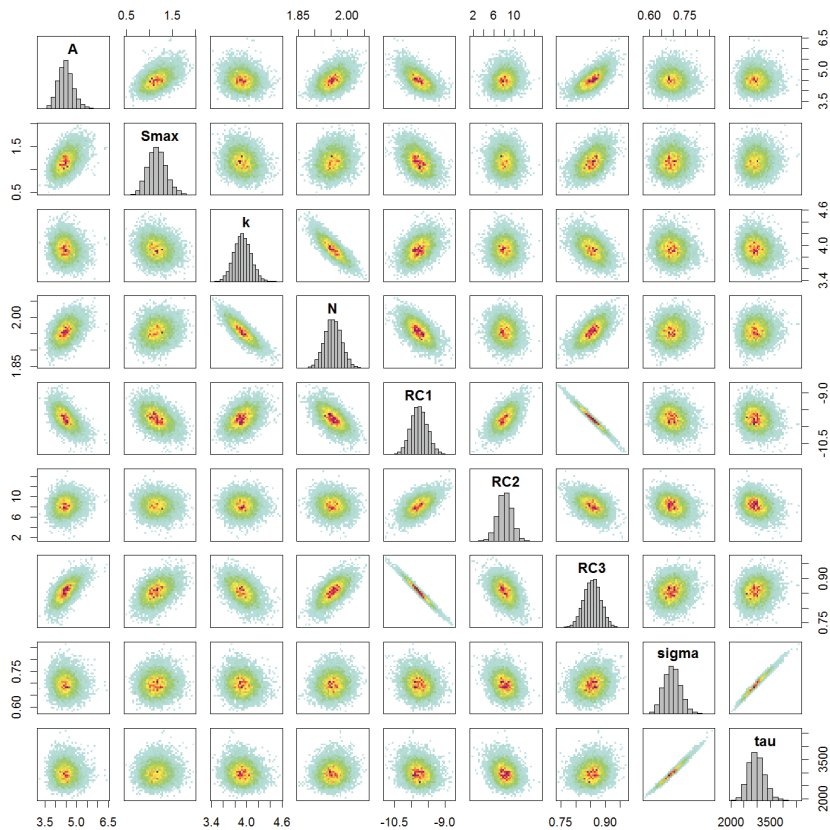

Fig. 5.10. Posterior parameters correlations in example II

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for remaining errors. In the example I it lumps structure deficits of the RR and measurement errors of streamflow, whereas in the example II the LEM compensates for structure deficits of the RL model, measurement errors of water levels and the uncertainty of the input variable i.e. precipitation. Thus, the direct comparison of inferred LEM in the two examples is not possible.

Parameter correlations

Mutual correlations between all parameters are presented in Figure 5.10. As can be seen, a strong correlation between all RC parameters was observed, what is reasonable for such empirical (fitted) parameters. Alternatively, if computation time is a limiting factor, one could consider reducing the number of the RC parameters to be inferred by keeping some of them at fixed values.

A visible correlation can be also noticed for two of the RR parameters, k and N, as it was previously observed in the example I in Sect. 5.2.1. A very strong correlation observed between the LEM parameters is rather intuitive because both parameters compensate together for the structural deficits of the RL model and all other uncertainty not explicitly acknowledged here i.e. water level measurement and rainfall input uncertainties.

Finally, a correlation between the RR and RC parameters is apparent. Intuitively, this can be explained by a mutual compensation of both submodels, RC and RR. As a consequence, inferred parameters of both submodels include these dependencies.

Fulfilment of statistical assumptions

Figure 5.11 presents a diagnostic analysis of RL model errors at the maximum of the posterior distribution based on the example of one event. Following the analysis discussed by an example of application to streamflow, the assumptions of i.i.d
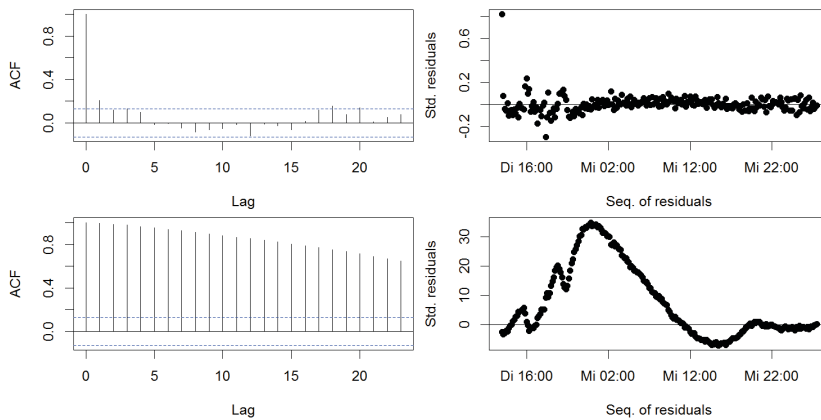
Fig. 5.11. Top panel, diagnostic plot of LEM residuals and, bottom panel, of residuals in the traditional Gaussian error model by the example of one event (No. 6) in example II. The left panels present sequences of residuals, bottom, and LEM residuals, top. A strong autocorrelation (ACF) can be observed for model residuals of Gaussian model, bottom right panel, whereas, ACF is significantly reduced for residuals of LEM model, top right

residuals are much better fulfilled for the residuals of the LEM than it could have been achieved with the traditional Gaussian error model.

5.3.2. Uncertainty of water level predictions

In the same fashion, the 95% PIs of water levels were approximated by means of Monte Carlo simulation with 1000 RL model runs by randomly sampling from the full posterior distribution and are presented in Figure 5.12.

The reliability of such derived PIs was assessed again by means of leave-one-out-cross-validation method in terms of their data coverage and sharpness. The corresponding coverage of the 95% PIs in validation equals 84.5% of validation data points, of which 14% and 1.5% lie above and below the upper and lower limits, respectively. The computed 95% PIs are on average 29.3% and up to 82.4% higher than the observed maximum values; assessed by the upper limits.

The justifiable extrapolation range for the RC is exceeded by the simulated water levels only for the event No. 19. This event nicely illustrates that extrapolating the RC beyond its justifiable range leads to unreliable predictions. The estimated PI are here clearly overestimated. Such a high water level as predicted by the RL model would most likely not occur in reality because of overland flow outside the flood plains. As this process cannot be modelled accurately with the applied RL, PIs are overestimated and this results in their poor coverage of observed data.

Thus, the computed PIs for predicted water levels are sharper than the ones computed for streamflows. However, because different variables are modelled, this finding cannot be directly interpreted.
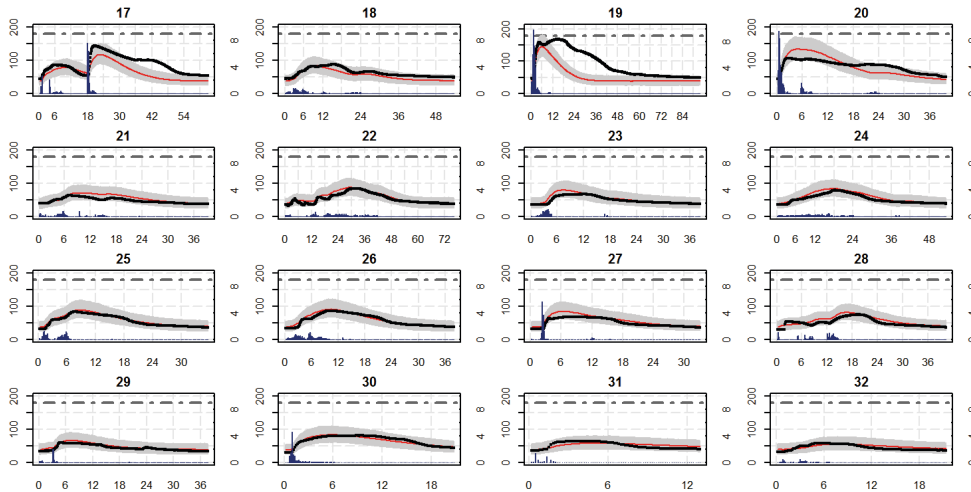
Fig. 5.12. 95% PIs for predicted water levels [cm] (grey polygons), left y-axes, in the Sluzew Creek during the validation. Bold numbers above label rainfall events; x-axes represent time from the beginning of the event; right y-axes – 10 minutes rainfall intensity [mm]. Dotted black line depict observations; red solid line – posterior mode; blue bars – observed rainfall; dashed grey horizontal line cuts a justifiable extrapolation range for the RC

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5.3.3. Contribution of the rating curve uncertainty

The contribution of the RC model parameters to the total PU, Sect. 5.3.2, was assessed by performing two additional independent MC simulations in analogy to the example I when input contribution was assessed. Thus, 95% conditional PIs were computed, first, while keeping the RC parameters at their posterior modes, and second, inversely the RR parameters were kept at their modes. By comparison of both, the importance of their contributions to the total PU was addressed and is presented for 16 events in Figure 5.13.
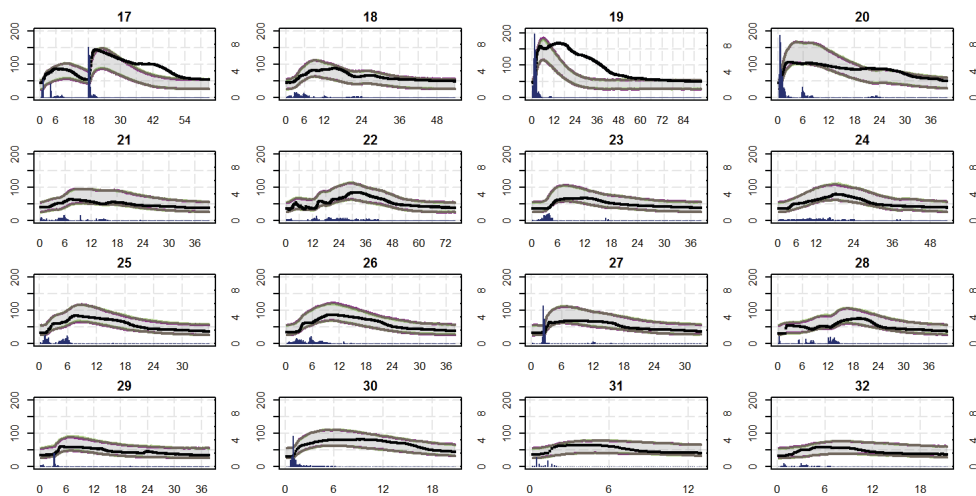

Fig. 5.13. Relevance of RC parameter uncertainty to the total PU, grey polygons, of predicted water levels [cm] in the Sluzew Creek. Overlapping lines depict 95% PI with ignoring uncertainty, green, in RC parameters and, brown, in RR parameters and point out a similar relevance of the RC and RR parameter uncertainties. Other notation as in Fig. 5.12

The corresponding PIs were found to be almost of the same relevance for both RR and RC parameters, both PIs lie close to each other and to the total PU limits (Fig. 5.13). On average both PIs contribute only about 3% to the total PU and a difference in their contribution is less than 1% (mean) in the validation mode, with a slight dominance of the RR parameter uncertainty. This finding would suggest, first, that the uncertainty in RR and RC parameters leads almost to the same PIs of water levels in the Sluzew Creek basin at this monitored cross section. Thus, for Sluzew Creek keeping parameters of both submodels at their modes would presumably lead to similar PIs as when the full posteriors for RR and RC are explored. Second, uncertainties of the RC and RR parameters contribute much less to the total PU in water level predictions while compared to the contribution of the LEM parameters alone (compare with grey polygons in Fig. 5.13).

5.3.4. Conclusions from the CASE STUDY II

The example II addressed the predictive uncertainty (PU) in hydrological modelling and in particular the uncertainty of calibration data for RR models which were assessed by means of the corresponding RC method. Based on the results from the case of Sluzew Creek, it can be concluded that:

- Only a part of the Sluzew Creek basin contributes to the surface runoff which is rapid during heavy rainfalls. This is in agreement with the findings in the example I. Uncertainty in water level predictions in Sluzew Creek, described as 95% PIs, are on average 29.3% and up to 84.3% higher than observations, assessed by water level peaks during the events. The contribution of the RR and RC parameters to the total PU was found to be almost of the same relevance in Sluzew Creek with a slight dominance of the RR parameters.

- Modelling water levels directly instead of streamflows appears to be beneficial because it allows one for incorporating the output uncertainty represented by the uncertainty in calibration data, and the RC parameters, into the modelling process and for assessing its contribution to the total PU.

- As it was already shown in the example I, the LEM significantly helps fulfilling statistical assumptions on errors of hydrological models. Using the simple Gaussian error model instead would clearly violate the assumption on i.i.d. residuals. The fulfilment is, however, not always perfect.

- Extrapolating the RC beyond the measurement range is usually a necessity for flood studies because flows measured in flood conditions are rare. However, extrapolating RC beyond the permissible range leads to unrealistic uncertainties of water level predictions which are usually overestimated. This is mostly due to the fact that RC, which was calibrated for streamflows within the stream channel, is used in the present events to compute flood flows which usually flow out of the stream channel onto floodplains. Thus, such flood flows are greatly reduced. The permissible range is, however, not necessarily equal to the measurement range, especially for irregular or complicated bathymetric profiles. Therefore, updating and validating rating curves should be crucial prior to modelling.

- The contribution of the RR and RC parameters to the total PU was found to be relatively small. The uncertainty of RR and RC submodels is, however, higher than this reflected only by their parameters uncertainty. The small contribution of the RR and RC parameters to the PU suggests that the largest contribution remains in the structural limitations of the RL model itself, lumped into the LEM. This requires, however, a careful interpretation because the RL consists of both RR and RC submodels. Finally, the presented approach to assess the contribution of output variable uncertainty in RR modelling was shown to be very practical. However, apart from pinpointing the output uncertainty contribution, it does not allow for direct modelling of streamflow. Thus, it is only useful when water levels are of interest, see further discussion in Sect. 5.5.

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5.3.5. Lessons learnt from practical applications of uncertainty analysis

Lesson I

Results from the Sluzew Creek basin suggest that i) the contribution of the input uncertainty to the PU is significant (up to 65%) and about 40% higher than the uncertainty of RR parameters themselves (up to 25%) (example I); ii) the contribution of RR parameters and output uncertainty, represented by RC parameters, to the total PU is almost of the same relevance with a slight dominance of the RR parameters (less than 1% difference) (example II). These findings are strongly case-related and require careful interpretation. Although the UA approach alone is transferable to other basins, the individual contributions remain always case-related. These two points are discussed further in Sect. 6.1.

Lesson II

The UA approach proposed in this thesis was proved to be beneficial in assessing flood predictions in SUBS. An explicit treatment of diverse uncertainty sources adds value to the analysis because it allows one to directly assess their contribution. In this, the rainfall multipliers and RC approaches to assess input and output variable uncertainties were shown to be very practical. Also the autoregressive continuous lumped error model (LEM) appeared to be sufficient in explaining the rainfall-runoff within the basin. Main limitations arise from methodological aspects of the study and are further discussed in Sect. 6.2.

Lesson III

The two explored application examples nicely demonstrated benefits of implementing uncertainty into hydrological model predictions. A direct confrontation of predictions computed, first, based on the mean value only and second, when the full posterior distribution is explored clearly showed that the possible risk of overflooding may be significantly underestimated in the first case. Several issues need, however, to be further discussed. Relativity high predictive uncertainties estimated in both examples intuitively rise the question of a practical value of such high uncertainty and a possibility of incorporating UA into decision making process. Results of the UA are given in terms of probabilities and these cannot be interpreted by most of the people. Thus, the added value of analysis may be lost if it is not properly communicated. The UA was tested with the use of simplified models, which are a common choice in flood studies in ungauged basins in Central-Eastern Europe. A relatively small contribution of RR and RC parameters suggests that the largest contribution remains in the structural limitations of the hydrological model itself, lumped into the LEM. This concerns doubts of the suitability of such simplified models used to flooding forecast. Finally, this also opens room for discussing the future perspectives of hydrological modelling.

6. DISCUSSION AND OUTLOOK
6.1. Interpretation of the results

6.1.1. Uncertainty contributions in application examples

Example I

It was found that the input uncertainty significantly contributes to the PU of RR models in the Sluzew Creek basin representing a typical SUB. On the one hand, a relatively large contribution proves a high variability in rainfall fields over the entire basin. Even if on average the observed rainfall at the rain gauges may be considered as representative for the SUB area, the variability observed in individual rainfall events was high. This variability is difficult to capture only by means of a sparse punctual gaining network (McMillan et al. 2011) and thus different methods should be sought (see further Sect. 6.3). On the other hand, rainfall multipliers ultimately increase the flexibility of the hydrological RR model and hence partly compensate for its structural deficits (Sikorska et al. 2012a). An increased number of calibrated parameters allows for a better model fit to the observed data. Thus, a relatively high contribution of the input uncertainty is likely also caused by mutual compensations of input error model and error model of RR structure deficits. This error is lumped together with the measurement error of streamflow into a LEM. It is further assumed that the measurement uncertainty of streamflow is much smaller than the model structure error itself and thus the LEM is dominated by the uncertainty in the model structure. However, this uncertainty cannot be decomposed from the LEM.

Example II

The parameter uncertainties of both the RR and RC submodels were found to be almost of the same relevance and much smaller than the remaining error represented by the LEM. The LEM lumps here model structure deficits of the RR and RC submodels, measurement errors of water levels and input uncertainty of precipitation. This relatively small contribution of RR and RC parameters to the total PU has two main reasons. On the one hand, all model parameters (RR and RC) can be defined precisely if the available recorded input-output data contain enough information. Thus, the parameter uncertainty may be reduced with more data. On the other hand, a simplified hydrological model applied here presumably produces a high systematic error in predictions. This error cannot be reduced with recorded input-output data because the model structure remains the same. Hence this error likely dominates beyond other uncertainty sources (Sikorska et al. 2013).

6.1.2. Generalization of example findings

The developed UA approach is general and can be applied to other studies. In contrast, the quantitative contributions of individual uncertainty sources found in this study cannot be directly transferred to other studies. Because of specific model structures, basin properties, available data and modeller's/expert's knowledge, the results of quantitative analysis always remain case related. Thus, to pinpoint individual uncertainty contributions for another study, the entire UA must be implicitly

performed for each research study. It is worth noting that quantitative results contain a subjective element due to the need to specify the prior on model parameters. Usually, this subjectivity will be mitigated by the data during the Bayesian inference as long as the specified prior allows for that; i.e. the prior assumed probability is not equal to zero for values for which there is evidence in data.

Another thing is that the results obtained for RR are based only on a short observation period, 3 years for RR. Thus, they can be strongly influenced by temporal conditions. Hence they do not allow for analysis of long term changes within the basin. For long-term analysis, more records should be gathered.

Although the merit of conceptual hydrological models lies in their conceptual parameters, which may be linked to basin properties and thus transferred to other studies without the need for direct model calibration. Because within the developed UA model parameters are described as a entire probability distribution and not as an individual parameters, they cannot be totally separated. This is due to the fact that inferred parameters include mutual dependencies and therefore lose some degree of their conceptual interpretation during the inference. Thus, they should be interpreted in term of the entire probability and only as the probability transfer to the other basins, e.g. as a prior, as also suggested by McIntyre et al. (2002).

6.2. Methodological aspects of the study and their limitations

The UA method developed within this work allows for assessing the importance of individual error sources. However, within this approach many decisions were made which need to be discussed in more detail. This includes in particular: i) using Bayesian inference for UA; ii) using rainfall multipliers; iii) rating curve (RC) uncertainty; iv) rainfall-water level (RL) model; v) lumped error model (LEM) and vi) strategies for uncertainty reduction.

6.2.1. Using Bayesian inference for uncertainty analysis

Bayesian inference was demonstrated to be very beneficial in assessing uncertainties of flood forecasting in SUBs. The great advantage of Bayesian inference lies in the direct interpretation of the PU, which truly represents the probability of model predictions. In the same manner, the best model prediction is the most probable estimate. Bayesian inference also allows one to easily incorporate prior knowledge into model parameters from various sources, such as experts' knowledge or previous studies. This is especially relevant for SUBs where typically no long-term recorded input-output data are available which would allow for a classical model calibration (Sikorska et al. 2012a). In addition, Bayesian inference has no high data requirements. Thus, already basic basin data enable a probabilistic statement on possible model estimates. Finally, Bayesian statistics allows for an explicit treatment of diverse uncertainty sources as input variables, model structure, parameters and output variables. This adds value to the analysis because it pinpoints important directions of further investigation in order to i) improve the accuracy of model predictions, and ii) reduce associated uncertainties (if reducible) (Sikorska et al. 2012a, 2013). In this way one can assess if e.g. more effort should be put into gathering more calibration

data for the selected hydrological model or whether another hydrological model sho-uld be chosen. The main limitation arises from the need to formally describe errors of a hydrological model and consequently to formulate the likelihood function.

6.2.2. Using rainfall multipliers

To account for the uncertainty in input rainfall, the rainfall multiplier appro-ach was adopted (example I). This approach was demonstrated to be very feasible in making predictions. However, it is limited to event-based modeling because an individual multiplier must be inferred for each RR event. Moreover, the approach fails if rainfall occurred but was not observed because a multiplier multiplied by a null value always yields null. As already pointed out in Sect. 6.1, using rainfall mul-tipliers increases model flexibility to reproduce observed input-output data. Thus, on the one hand, inferred rainfall multipliers will reflect this dependency. Consequently, inferred rainfalls should be treated as inferred rather than real (Seibert, Beven 2009; Sikorska et al. 2012a). On the other hand, uninformative prior on input uncertainty would presumably lead to overestimating its contribution and therefore should be avoided (Renard et al. 2010, 2011). The approach of rainfall multipliers could also be extended to account for uncertainty in other variables as e.g. streamflow. This, ho-wever, is not straightforward to implement because streamflow cannot meaningfully be divided into events. Thus, further research is required in this regard.

6.2.3. Rating curve (RC) uncertainty

The relevance of output uncertainty for RR models was assessed by quantifying the uncertainty in RC parameters (example II). The uncertainty of the RC submodel itself cannot, however, be assessed because the structure error of the RC is insepara-bly lumped into the LEM. To quantify the total uncertainty of the RC submodel, its structural error needs to be decomposed from the LEM and explicitly acknowledged. To this end, further research would be required.

6.2.4. Rainfall-water level (RL) model

Incorporating RC into the RL model allows one to assess the contribution of calibration data uncertainty for RR models (example II). However, it does not al-low for direct modelling of streamflow, which is an internal state of the RL model. Thus, only water level can be modelled. To estimate streamflow, water level must be converted through the RC along with the accompanying uncertainty. Alternatively, streamflow could be modelled directly. This would require an explicit treatment of the RC submodel.

6.2.5. Lumped error model (LEM)

Bayesian inference requires an explicit treatment of hydrological model errors. To this end, a LEM was applied to lump together all uncertainty sources not expli-citly acknowledged. Thus, the LEM lumps different errors in the two application

examples and consequently specific findings from those two studies cannot be quantitatively compared.

In both examples, the LEM proved to be sufficient in explaining the rainfall--runoff processes with the basin because it accounts for the correlation apparent between errors of hydrological model predictions. Thus, it appears to be a promising alternative to the commonly applied classical Gaussian independent error model.

6.2.6. Strategies for the predictive uncertainty reduction

The developed uncertainty analysis allows pinpointing the importance of individual error sources. This information is relevant for planning strategies of uncertainty reduction. In this regard, a few recommendations can be given.

Model parameter uncertainty can be gradually reduced by gathering more measured input-output data and using more precise information to elicit prior i.e. more accurate maps or remote sensing data. The reduction of the PU is, however, limited due to other source contributors, and after reaching a certain point not noticeable any more.

Input rainfall uncertainty may be reduced with more precise rainfall information. One could greatly benefit with a denser network of rain gauges, weather radar or retrieving data from telecommunication networks such as microwave links (Fencl et al. 2012; Bianchi et al. 2013). A denser rain-gauge network typically has some costs attached to it, such as equipment purchase and maintenance. In contrast, using radio links, one may benefit from already existing infrastructure over all continents and a high resolution of links (locations). Thus, no additional costs for equipment have to be borne.

Output uncertainty of RR models could be reduced with more precise information on a RC used to derive calibration data for RR models. This may be especially significant for poorly gauged cross sections with only a few measured water level-streamflow records available or for dynamic basins where cross sections significantly change seasonally or with changing land use. Uncertainties on already existing RCs could be reduced by incorporating remote sensing data from satellites (Di Baldassarre, Uhlenbrook 2012). Finally, it is crucial to update RCs frequently and to successively extend their measurement range to also cover flood conditions (Domeneghetti et al. 2012).

Alternatively, one could consider improving the structure of a hydrological model. This will reduce systematic errors observed in predictions; structure model error. In general, this is tied to the availability of input and output data, and their content, to calibrate parameters of the improved model. If data do not contain enough information to infer all model parameters, the parameter uncertainty will increase more than the uncertainty of model structure decreases.

6.3. Outlook
6.3.1. Practical value of the predictive uncertainty

Quantification of the predictive uncertainty in hydrological modelling is relevant for many studies, such as flood hazard analysis, water management, mitigation strategies or urban development analysis (Montanari 2007; Efstratiadis, Koutsoyian-

nis 2010; Ramos et al. 2010). For instance, UA could be useful to evaluate whether a bridge, floodplain or city will be overflooded during the forecast rainfall event and with which probability. For water quality analysis not only temporal but rather long--lasting conditions are important. This is due to the fact that many aquatic species may endure a relatively high dose of pollution if it is only temporary but will extinct with a smaller long-lasting dose. In this, uncertainty on water quality estimates may support water quality management by formally (probabilistically) comparing diverse strategies for water quality improvement and their impacts on receiving waters.

UA quantifies the model PU under current conditions e.g. the current knowledge on the future or current state. This knowledge may be (very) uncertain and this will be reflected by large uncertainties. This may lead to concerns of usefulness of such large uncertainties. Intuitively, uncertainty of predictions referring to unknown conditions cannot be expected to be small. Thus, PU of 100% of high flows and 1000% for low flows are not uncommon and have been reported [Blöschl, Montanari 2010]. In this, uncertain information, even if not preferable, is always more useful than a false certainty which may pose catastrophic consequences (Di Baldassarre et al. 2013). For instance, if one is not aware of the possible risks, prevention or mitigation strategies cannot be even considered.

6.3.2. Incorporating UA into decision making process

A decision making process weighs different options by social preferences. In this, UA of hydrological predictions can support the decision making process in three ways (Reichert 2012). First, UA integrates scientists' knowledge of (un)certainty and possible risks and social responsibility of predictions. Providing decision makers with information on PU is the basis for risk evaluation and may lead to other decisions than without consideration of uncertainty. Second, interpreting model outputs in terms of a random variable instead of certain single values allows for a better analysis and comparison of diverse scenarios by means of diverse criteria that are not only limited to a flat interpretation. Third, probabilistic quantification of PU allows one to hierarchically order different scenarios by diverse goals and socio-economic circumstances and in accordance with uncertainty levels, e.g. risks of dam breaks or levees overflowing for different strategies considered. This is especially important for urbanized sites, where the socio-economic consequences of even moderate potential flooding can be severe (Aronica et al. 2013). Lastly, there is an increasing need for a better link between science and practice in the discipline of hydrology so that UA does not only remain the scientists' issue but becomes a routine procedure in water management. This is already standard in ecological, medical and general risk analysis.

6.3.3. Communicating the uncertainties

Interpreting model outcomes in term of probability distributions instead of single outcomes may pose a problem, especially for decision makers who are usually not statistically trained. In a classical (deterministic) approach a model provides only a single prediction, which is much easier to understand for most people. For instance, the bridge will be flooded or not and the predicted flood flow can be 'exactly' es-

timated. This information may, however, lead to overconfidence because a possible risk is underestimated, as shown in this thesis. Because many people feel uncomfortable with a probabilistic statement (Todini 2007), it has become common practice to communicate probabilities in terms of frequencies. For instance, one could communicate a possible flood risk in two ways; first, as an annual exceeding probability of 1%, and second, as a flood flow which occurs once per 100 years on average. The first information is hardly understandable for many people. The second information can be more easily understood but may lead to false confidence. A person may interpret it as follows; if the flood with a return period of 100 years already occurred in his life, it will definitely not occur again. However, an occurrence of 1%-flood flows is theoretically possible every year. This is not understandable by people without a statistical or mathematical background. The information on a possible risk, even if complete, becomes useless if it cannot be interpreted or understood. Consequently, more effort should be put in a communication of uncertainties to decision makers.

6.3.4. Hydrological modelling in Central-Eastern Europe

Hydrological modelling in Central-Eastern Europe, e.g. Poland, is especially difficult because conditions of basins located in this part of Europe are expected to rapidly change in the future due to foreseen urban growth (EEA 2006). Thus, not only climate but also social changes must be forecast, and these are difficult to accurately predict. Consequently, hydrological models with parameters calibrated to current conditions will most likely not well predict the basin response under changed conditions. Thus, the benefit of using complex models with numerous fitted parameters may be lost (Blöschl, Montanari 2010). In addition, due to economic costs and practical problems, it is usually not possible to fully cover all basins being at flood risk with a proper monitoring program; e.g. continuous measurements of rainfall, streamflow and water level. Especially SUBs remain ungauged due to a relatively low flood risk, which is understood as a compromise between costs put into flood prevention and possible damages in case of flooding. To assess flood risk and associated water quality problems under the changing environment of a basin, flexible models which can be easily adjusted to new conditions are therefore sought. For this reason, simple conceptual models, despite their limitations, are justified because it is straightforward to derive their parameters and predictions for different future scenarios i.e. under changing conditions of a basin (climate and social). The advantage of such models over structurally more complex models is a limited number of conceptual model parameters which can be inferred independently from recorded data (see above Sect. 6.1). Hence, classical model calibration can be avoided. Next, due to their reduced complexity, such models capture only the most important processes involved in the RR process being modelled. Thus, the RR process is usually reduced to model only direct streamflow without considering other components of water balance such as ewapotranspiration or groundwater flow. Finally, such models are not data demanding. Short series of recorded input-output data gathered under current conditions are usually enough to improve predictions of such models and reduce their uncertainty due to model parameters.

6.3.5. Applicability of simplified models to flooding forecast

The proposed UA approach was tested on conceptual models with a reduced complexity. As shown by results from the examples, such simpler models can satisfactorily predict flood events. Thus, they are useful for flood predictions in poorly gauged basins with only a few recorded input-output data available or when assessing effects of future changing conditions (climate or social). However, all models are limited in predicting extreme conditions where unforeseen interactions occur, i.e. external processes that are not included in the model structure (Sikorska et al. 2013). If a model is flexible enough, its parameters may be adjusted during the calibration to fit observed patterns. In contrast, if a model has a reduced complexity and flexibility, it may not be possible to match the calibration data. Usually, this also results in large PU bands that are dominated by structural limitations of hydrological models, which cause large systematic errors in predictions, see Sect. 6.1. This may raise concerns regarding the practical applications of these types of models. This uncertainty is not reducible with more recorded data because the model structure remains the same. If this is a crucial point, one should consider different models with a more complex structure. However, even if more details are included into the model structure, usually it is not possible to reduce all PU. Some uncertainty always remains due to the variability of the basin being modelled (Aronica et al. 2013). Interestingly, more complex models may not necessarily provide smaller uncertainties because the parameter uncertainty increases.

6.3.6. Perspectives of hydrological modelling

In the future, hydrological modelling is foreseen to further develop in three main directions: i) improving hydrological predictions under changing conditions; ii) improving real-time forecasting; iii) extending available data and extracting their content. This work covers the first issue only.

Hydrological modelling under changing conditions (climate and social) of the basin should adapt to these conditions and therefore cannot be considered stationary (Montanari et al. 2013). Thus, there is a need to move from deterministic modelling towards stochastic modelling. This can be achieved by including an uncertainty element in predictions of already available deterministic models. In this, to provide more accurate information on possible system states, new methods should be searched for to improve model structure, model predictions and reduce their uncertainties. Alternatively, hydrological modelling may profit from developing stochastic models which describe the basin process as a stochastic process. Such models rely on statistical information and thus may better imitate stochastic patterns of the original system. These models still require formulation of predictive uncertainty components, as input, parameters or output.

In terms of assessing the risk arising from current hydrological conditions e.g. of flooding, further development of real-time forecasting is required. Model predictions in real time can usually benefit from less uncertain information because it is easier to accurately predict a system state in e.g. one hour or one day than in one or ten years.

However, some uncertainty is always present. To assess this uncertainty, real-time forecasting requires flexible models and UA techniques, whose parameters can be easily adjusted due to changed conditions (Romanowicz, Beven 1998). Some techniques are currently available, e.g. Kalman filter or particle filter, in which the uncertainty in state variables is propagated through time so that the model's estimates are continuously updated with the observed information (Kalman 1960). This is conceptually similar to Bayesian updating (e.g. Chen 2003). To be efficient in practice, estimates from real-time forecasting should ideally reach an interested group in as short time as possible. In this, the early warning system could greatly benefit from virtual social groups, which are becoming more and more popular (e.g. facebook, twitter). Nowadays mobile phones with internet access ('smart phones') are becoming a standard device in citizens' everyday lives. Thus, warning information on flood risk spread via social networks can potentially reach recipients faster than e.g. radio or television.

Finally, the problem with the available data still remains unresolved. In this, the increasing importance of open access to information should be used to a greater extent. Fast developing new techniques and technologies in engineering and monitoring can support hydrological modelling by providing more data and of a better accuracy e.g. GIS data, remote sensing and with a better resolution (Chormański et al. 2011; Schumann et al. 2009). Attention should also be paid to extracting more information from already existing data. Important information on previous states of the basin, stream or flood flows or long-term changes can be gained from potentially unrelated disciplines, such as e.g. historical information from reports, pictures, newspapers. Also the rapidly increasing popularity of social networks (e.g. facebook, twitter) could open new possibilities to obtain meteorological and hydrological. With no additional monetary costs much can be gained from e.g. flood observer groups, which can provide detailed and accurate information on current hydrological states around the world such as water levels, which in turn can be utilized in flood hazard analysis or to calibrate models a posteriori.

7. FINAL CONCLUSIONS AND RECOMMENDATIONS
7.1. Final conlusions

Many research questions were raised in the introduction. Several of those could be answered but some are still open and need further investigation.

1. The uncertainty analysis (UA) developed has been demonstrated to be a feasible method to assess the predictive uncertainty of rainfall-runoff (RR) models in small urbanized basins (SUBs). The UA is based on the Bayesian inference. The UA provides uncertainty bands that have a probabilistic interpretation. Moreover, it allows one to pinpoint the relevance of individual sources contributing to the total predictive uncertainty (PU). The approach is broadly applicable to studies in other basins and with other hydrological models. The main limitations arise from the need to formally describe errors of a hydrological model and consequently to formulate the likelihood function.

2. Errors of a hydrological model have been shown to be heavily autocorrelated. This correlation cannot be represented by a classical independent Gaussian error

model, which has been typically applied in hydrological modelling. In this regard, the autoregressive lumped error model has been proven to be a promising alternative.

3. The input rainfall uncertainty has been demonstrated to be strongly time-dependent. To capture this variability, an approach with time dependent parameters is needed. In this work, the adopted rainfall multipliers approach has been demonstrated to be practical in describing errors of input rainfall to RR models.

4. The output uncertainty of RR models, which is streamflow, has been explored by the parameter uncertainty of the corresponding rating curve (RC). This approach has been demonstrated to be useful to assess the contribution of the output streamflow uncertainty. However, it does not allow for direct modelling of streamflow.

5. The practical value of quantifying prediction uncertainties lies in supporting water management through providing decision makers with a broader basis for scenarios and strategies analysis. In addition, quantifying individual uncertainty source contributions allows one to plan strategies for reducing the predictive uncertainty. Reduction of the predictive uncertainty is generally limited due to the model structure and availability and information content of input-output data to infer model parameters. Incorporating UA into practical applications remains, however, a challenging task. Most of all, this is because many people are not statistically trained to interpret and understand the output information from UA. More efforts in teaching decision makers and lay people on how to handle and understand this additional dimension of information would be desirable.

6. To predict the consequences of future (climate or social) changes, one has to rely on models in which parameters can be easily adjusted. Unfortunately, complex models usually require calibration against observed input-output data in order to provide reliable predictions. This cannot be achieved for future unknown conditions. Models with a simplified structure therefore remain an attractive option because they can provide predictions based only on short-term data. Their predictions, however, are usually connected with high predictive uncertainty.

7. Simplified models also often remain the only feasible tool to provide hydrological predictions in Central-Eastern Europe. Because this part of Europe is at high risk of future social changes such as urban growth, it is difficult to foresee future basin conditions. Moreover, the existing monitoring programs do not cover all basins at risk of flooding or water quality problems and is usually limited to infrequent measurements only. All this makes hydrological modelling in Central-Eastern Europe especially challenging.

8. The uncertainty of RR predictions in Sluzew Creek has been found to be large and dominated by hydrological model structure uncertainty and input variable uncertainty (rainfall). The first source cannot be effectively reduced by gathering more input-output data if the model structure remains untouched. If it is a critical point, the hydrological model should be improved. Alternatively, input rainfall uncertainty could be reduced with more accurate information on observed rainfalls. Output and parameter uncertainties were demonstrated to be less important in this basin.

7.2. Recommendations for future research

In the course of this work, several simplifications and assumptions were made and some issues were neglected. These issues require further research.

• Structure errors of hydrological models have been suggested to be an important source of the total predictive uncertainty of simplified models. However, its direct quantification was not possible. Thus, further research should aim at an explicit treatment of this error and its decomposition from other uncertainty sources. A promising approach to formally describe model bias (model structure error and input error) has been recently tested (Del Giudice 2013; Honti et al. 2013). This will require an explicit treatment of all uncertainty sources.

• Also the high contribution of the input uncertainty in RR modeling needs further investigation. In this regard, it would be interesting to investigate how much the input rainfall uncertainty can be reduced with more precise rainfall information such as from radio link networks or radar data. In particular, one could explore how the rainfall information from radio links can reduce the total predictive uncertainty of RR predictions in Sluzew Creek. Within this approach observed rainfall is computed from the information on radio links attenuation due to rainfall drops impact. To extract rainfall information from radio link networks, a promising technique has been already developed and is being tested (Bianchi et al. 2013).

• The small contribution of the output uncertainty represented by the RC parameters in the total predictive uncertainty for RR models also requires further research. The relevance of the entire RC method uncertainty in the RR model predictions uncertainty could be especially interesting for further investigation. A promising approach to quantify the RC method uncertainty has been developed (Claps, Di Baldassarre 2011). To include this uncertainty in hydrological model predictions, further investigation is needed.

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Summary

Analiza niepewności prognozy wezbrań opadowych w małej zlewni zurbanizowanej: W celu oceny wpływu zmieniających sie warunków zlewni na ryzyko powodziowe i jakość wody oraz opracowania strategii łagodzących, planiści i decydenci potrzebują prognoz modeli hydrologicznych. Prognozy te obarczone są zwykle istotną niepewnością o różnym źródle. W ramach tej pracy opracowano kompletną Bayesowską metodykę do analizy niepewności w modelowaniu wezbrań opadowych w małych zlewniach zurbanizowanych. Metodyka ta jest niezależna od charakterystyk zlewni badawczej i modeli hydrologicznych. Oryginalnością pracy jest połączenie innowacyjnych metod do opisania niepewności wejścia, parametrów modelu, struktury modelu i wyjścia modelu. Przydatność analizy niepewności wykazano za pomocą dwóch praktycznych badań, w których modelowano natężenie przepływu oraz stan wody, i zweryfikowano na zlewni Potoku Służewieckiego w Warszawie. Wyniki analizy pokazały, iż niepewność predykcji w tej zlewni jest duża i zdominowana przez niepewność wejścia i struktury modelu. Głównymi ograniczeniami metody jest konieczność formalnego opisania błędów struktury modelu oraz sformułowanie funkcji wiarygodności niezbędnej do przeprowadzenia Bayesowskiej analizy niepewności.

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Glossary of abbreviations

B	model Bias (model structure and input uncertainty)
E	model errors (model structure, input and measurements uncertainty)
ε	measurements (output) uncertainty
Em	model errors without input uncertainty (model structure and measurements uncertainty)
EP	effective precipitation
g	forward transformation
g^{-1}	backward transformation
i.i.d.	identically and independently distributed (errors)
l	predicted deterministic water level
L	predicted stochastic water level
LEM	lumped error model
Lo	observed water level
LR^{-1}	inverse of water level-runoff (model)
M	deterministic hydrological model
q	predicted deterministic streamflow
Q	predicted stochastic streamflow
Qo	observed streamflow
p	probability distribution
P	probability
P	precipitation
pdf	probability density function
Po	observed input precipitation posterior posterior probability distribution
prior	prior probability distribution
PU	predictive uncertainty
Px	transformed observed input precipitation
R	input rainfall error (model)
RC	rating curve (model)
RL	rainfall-water level (model)
RR	rainfall-runoff (model)
SUB	small urbanized basin
θ	model parameters
UA	uncertainty analysis
X	model input variable
Xo	observed model input variable
Xx	transformed observed model input variable
y	predicted deterministic model output
Y	stochastic model output variable
Yo	observed model output